



# HOW TO CHARGE LIGHTNING

The Economics of Bitcoin Transaction Channels

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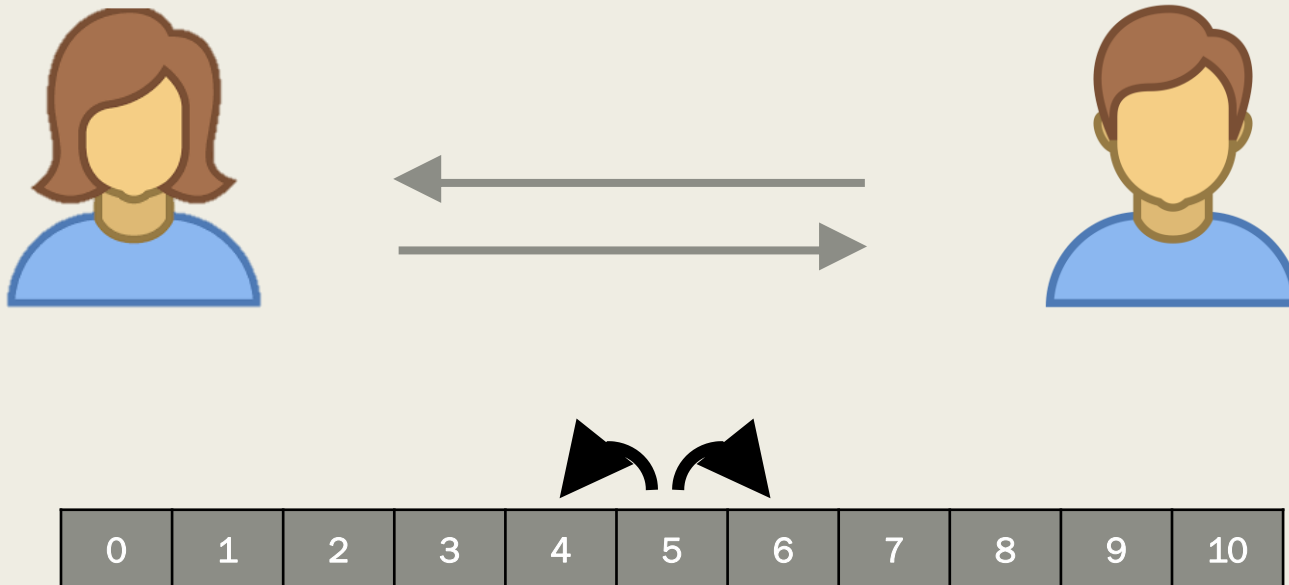
# Goals

We want to understand

- How to manage payment channel networks
  - *(channel funding, when to reset them, etc.)*
- The effect of channels on the economics of Bitcoin
  - *on-chain & off-chain volume and fees*

We lack data. Requires lots of assumptions on transaction size distributions, usage patterns.

# Managing a single channel

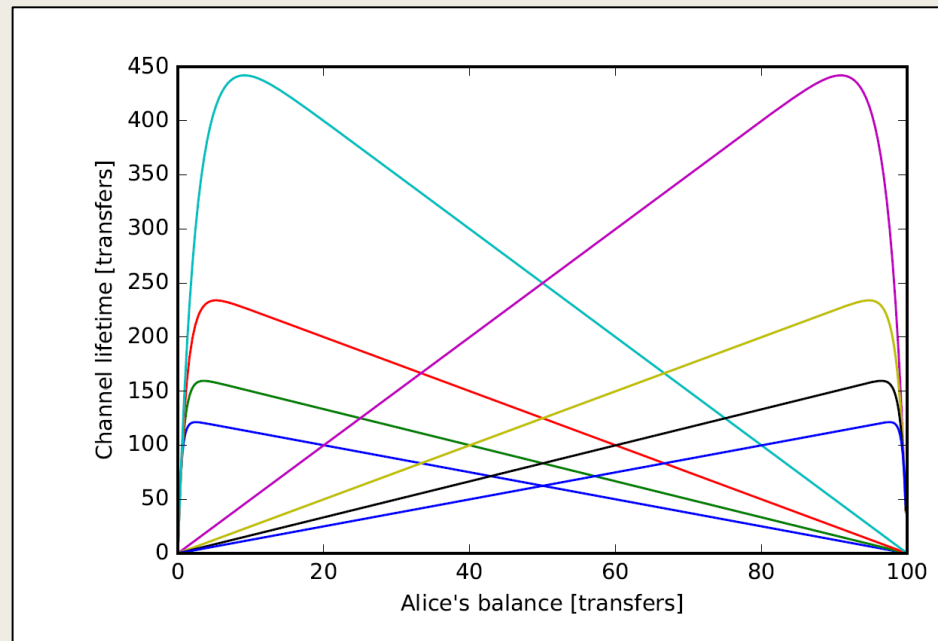


We assume a random walk (biased or unbiased) on the channel state

# Basic facts about single channels

- If Alice sends 1 unit to Bob at Poisson rate  $\lambda_A$ ,
- Bob sends 1 unit back at rate  $\lambda_B$ ,
- there is  $w$  units of capacity in the channel,
- $m$  in Alice's hands,  
expected channel lifetime:

$$\tilde{X}_m = \frac{m}{\lambda_A - \lambda_B} - \frac{w}{\lambda_A - \lambda_B} \left( \frac{1 - \left(\frac{\lambda_A}{\lambda_B}\right)^m}{1 - \left(\frac{\lambda_A}{\lambda_B}\right)^w} \right)$$



# Basic facts about single channels

- If Alice and Bob exchange 1 coin with the same probability, then they will do

$$X_m = wm - m^2$$

transfers in expectation before hitting the boundary.

- So, if we start a channel funded equally, its expected number of transfers is:

$$\text{Transfers}(w) = \frac{w^2}{4}$$

# How much do people transact?

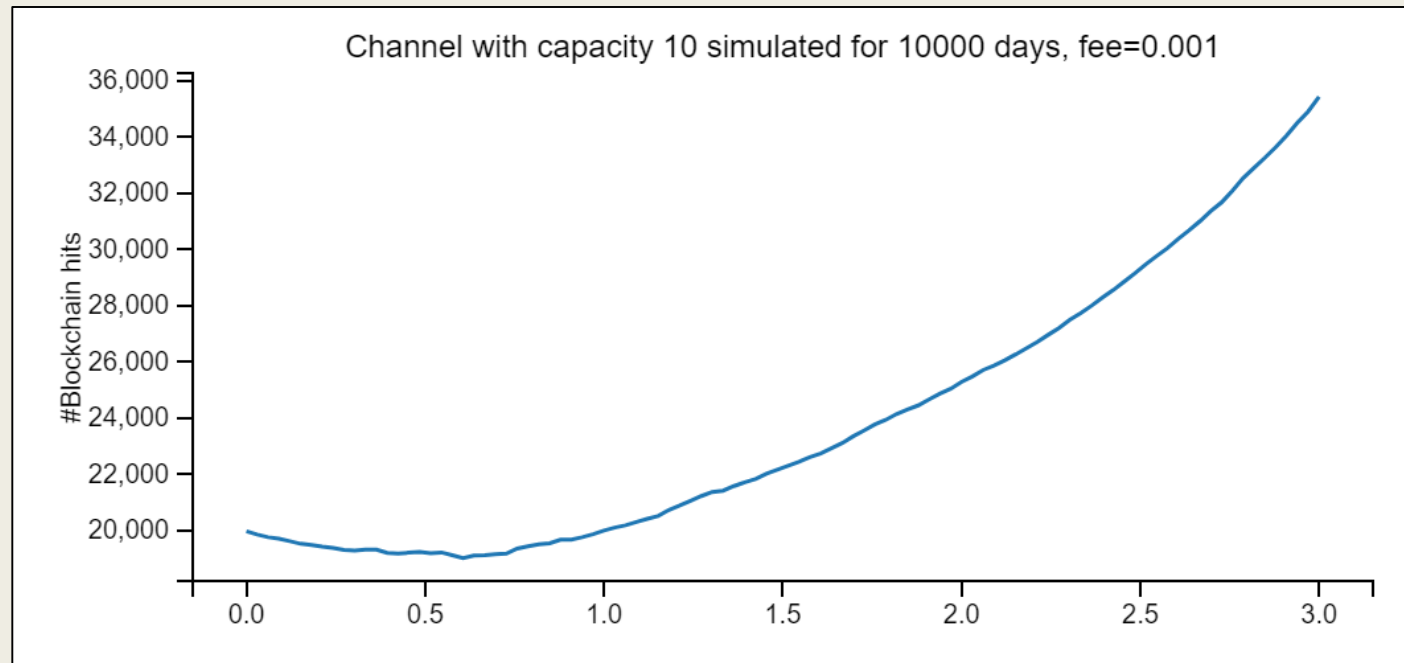
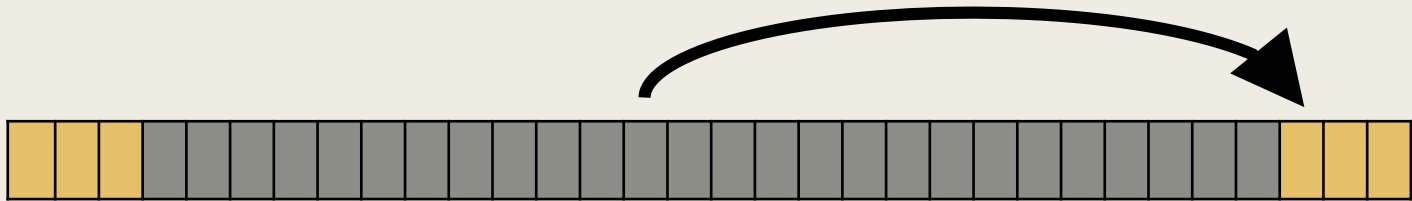
- We make small payments often
- We make large payments infrequently
- Data usually shows: power law behavior, e.g.

$$\textit{fraction of payments} \propto \frac{1}{\textit{amount}^2}$$

# Channel resets

With varying payments sizes:

- When we count the number of blockchain hits, it is profitable to reset the channel near the border



# How much do channels cost?

Two main costs:

- Setup and settlement cost of channels (blockchain fee)
- Interest rate payments to fund the channel itself

## Fees

- Fixed fees on the blockchain
- But on lightning, larger transactions shorten channel lifetime more. Charge by transfer volume.
- At least to cover costs

## Choice

- Txs choose cheapest route (lightning / blockchain)
- Or no route if both are too expensive



# Steps in our approach

- Pattern of payment flows and distribution of amounts
- Channel management (topology + funding + reset policy)
- Market equilibrium for fees

# Some intuition

- Large transfers prefer the blockchain (fixed fee)
- Small transfers split the cost of channel creation

Both compete for blockchain records.

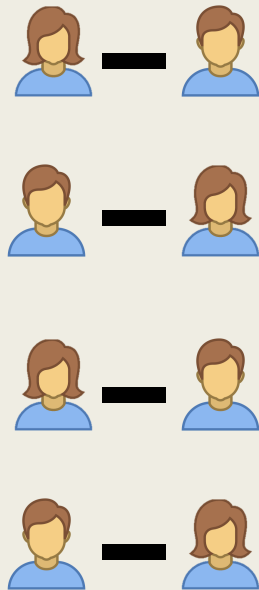
# Parameters

(for running example)

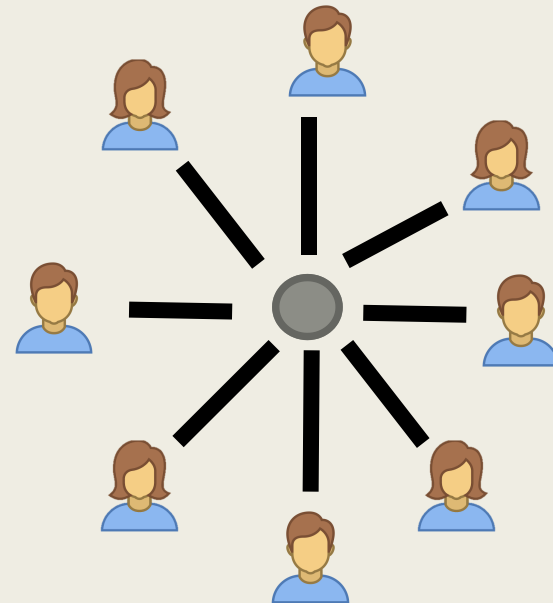
- Each person does 10 txs per day (in expectation)
  - *Drawn from a power law distribution.*
- Willing to pay 1% of transfer size as fees
- 4% yearly interest rate
- 288,000 on-chain records per day (pre segwit)

# Topologies

Pairs



Uniform Payments



(with central hub)

Hub: Lots of flexibility, double the channel costs

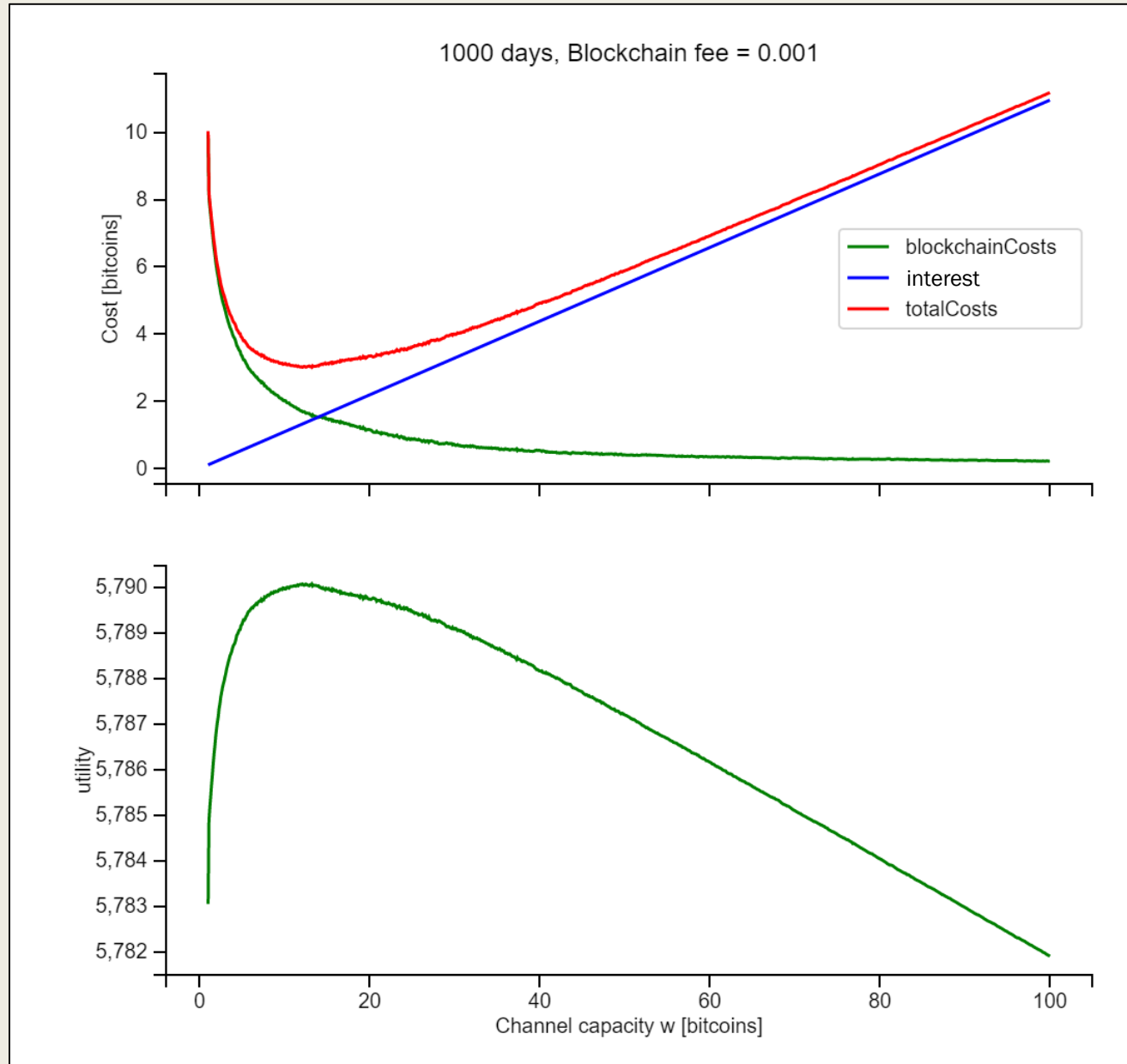
# Walkthrough example (in the pairs model)

- Given the on-chain fee we derive:
  - *Optimal reset radius*
  - *Optimal channel funding*
  - *Fee for lightning transactions*
  - *Demand for blockchain records at that price*
- Then find market clearing fees that sell exactly all blockchain records.

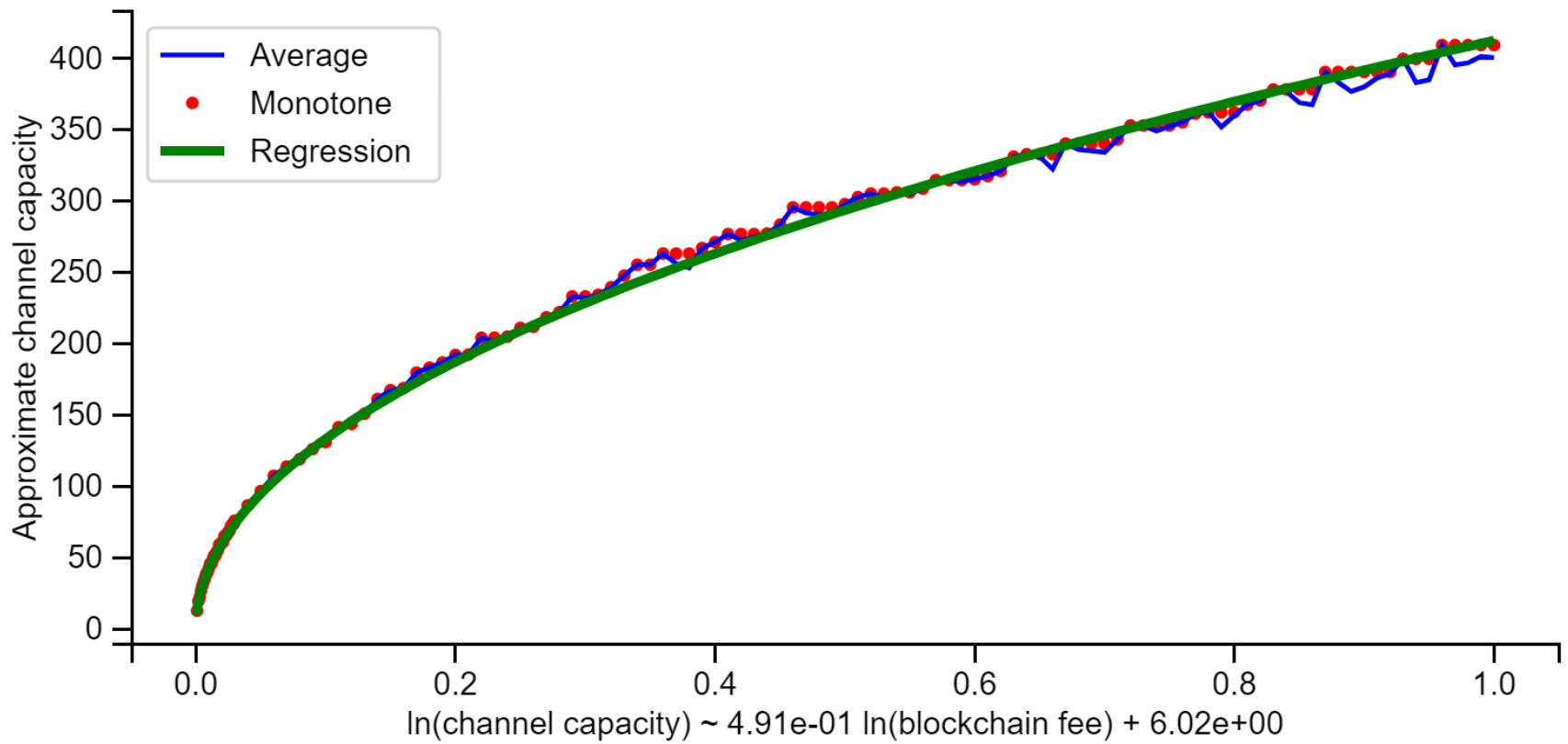
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# BEHAVIOR OF INDIVIDUALS

# Funding the channel



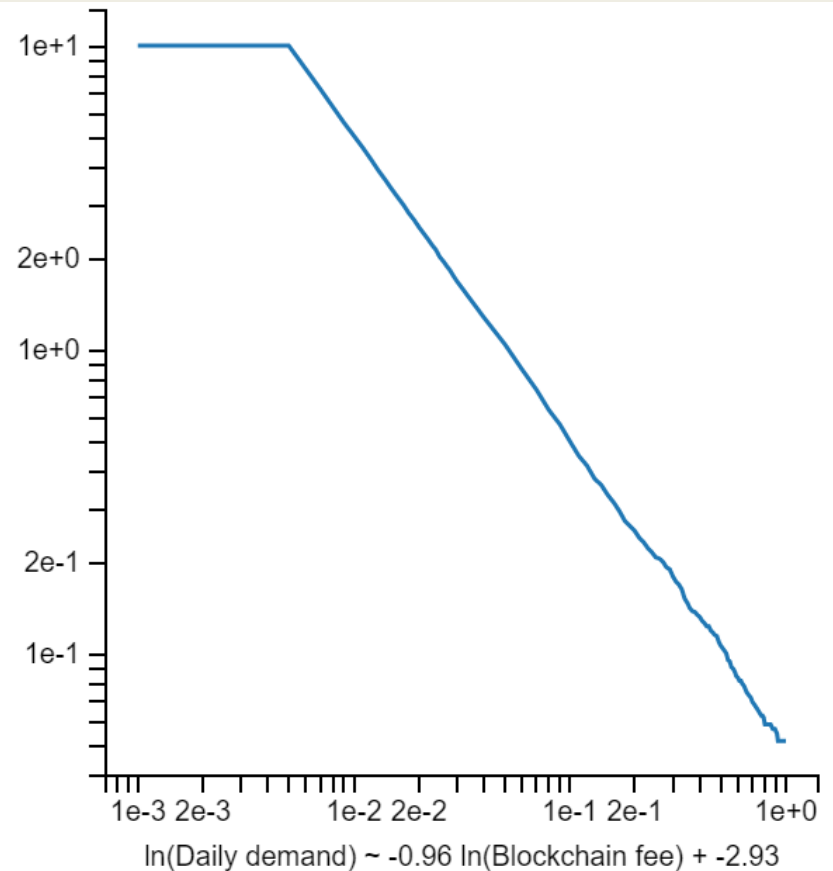
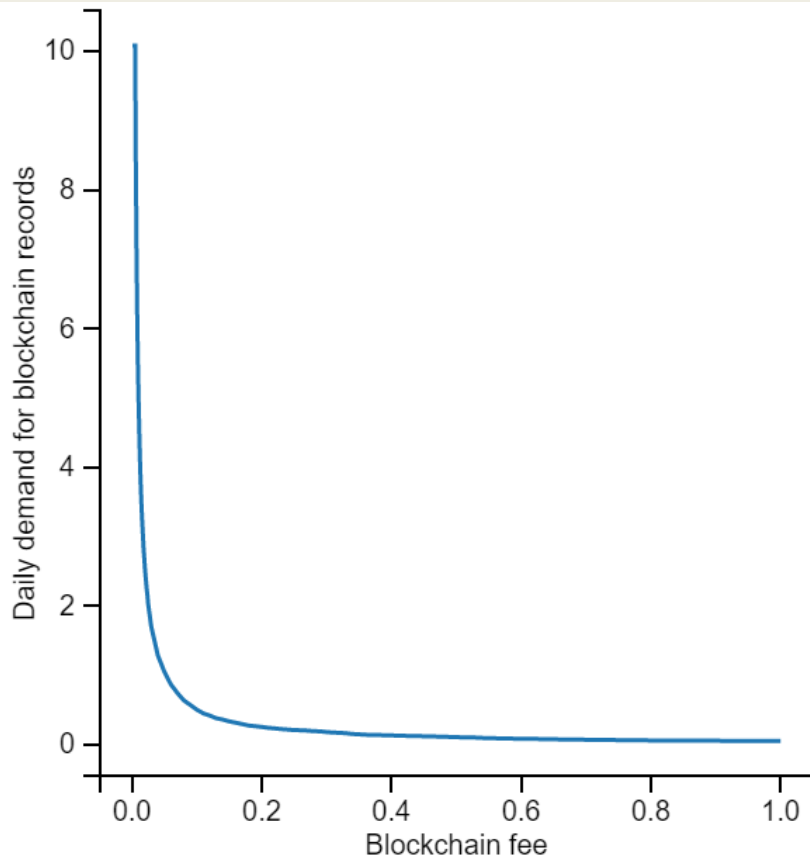
- Channel capacity grows proportionally to  $\sqrt{fee}$
- So do interest payments





# Demand for blockchain records

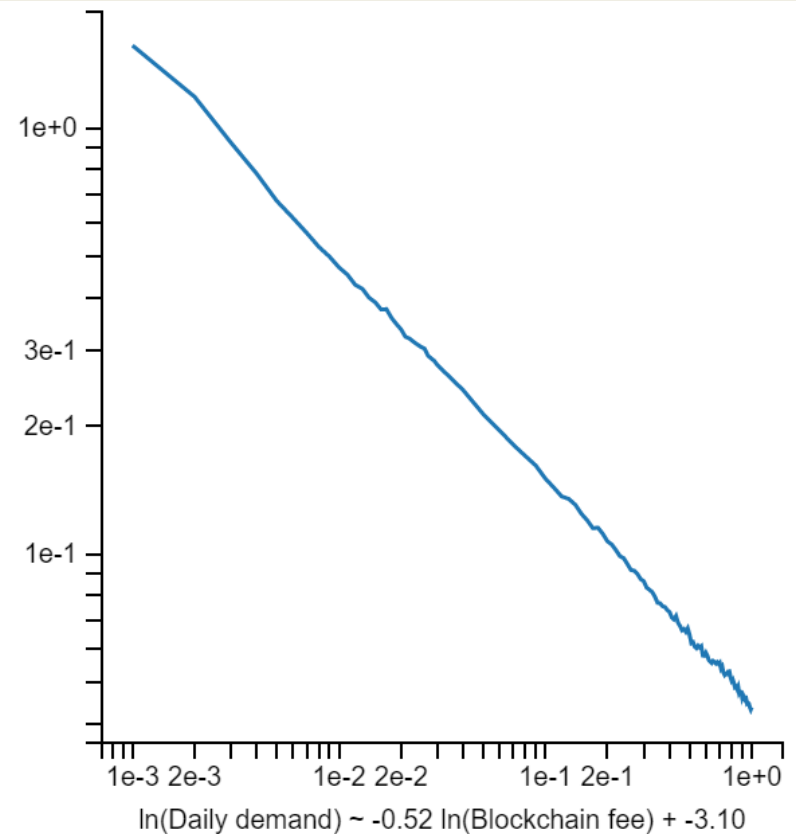
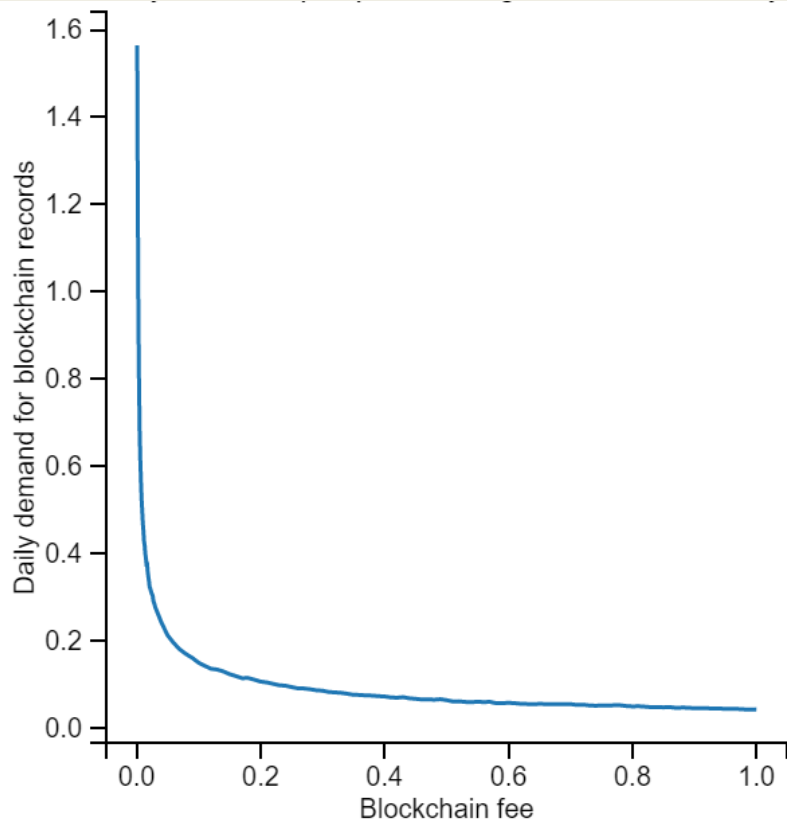
In a world without lightning: demand is  $\propto \frac{1}{\text{fee}}$



# Demand for blockchain records

In a world with lightning:

■ demand  $\propto \frac{1}{\sqrt{\text{fee}}}$





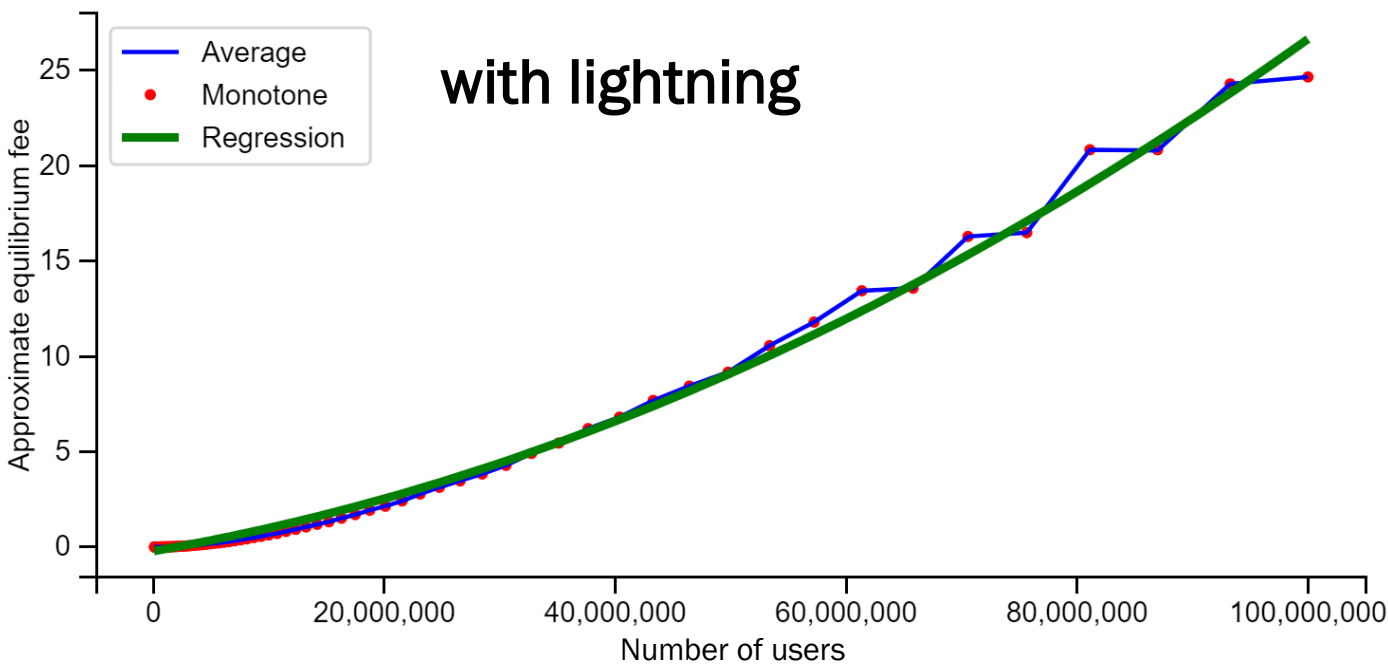
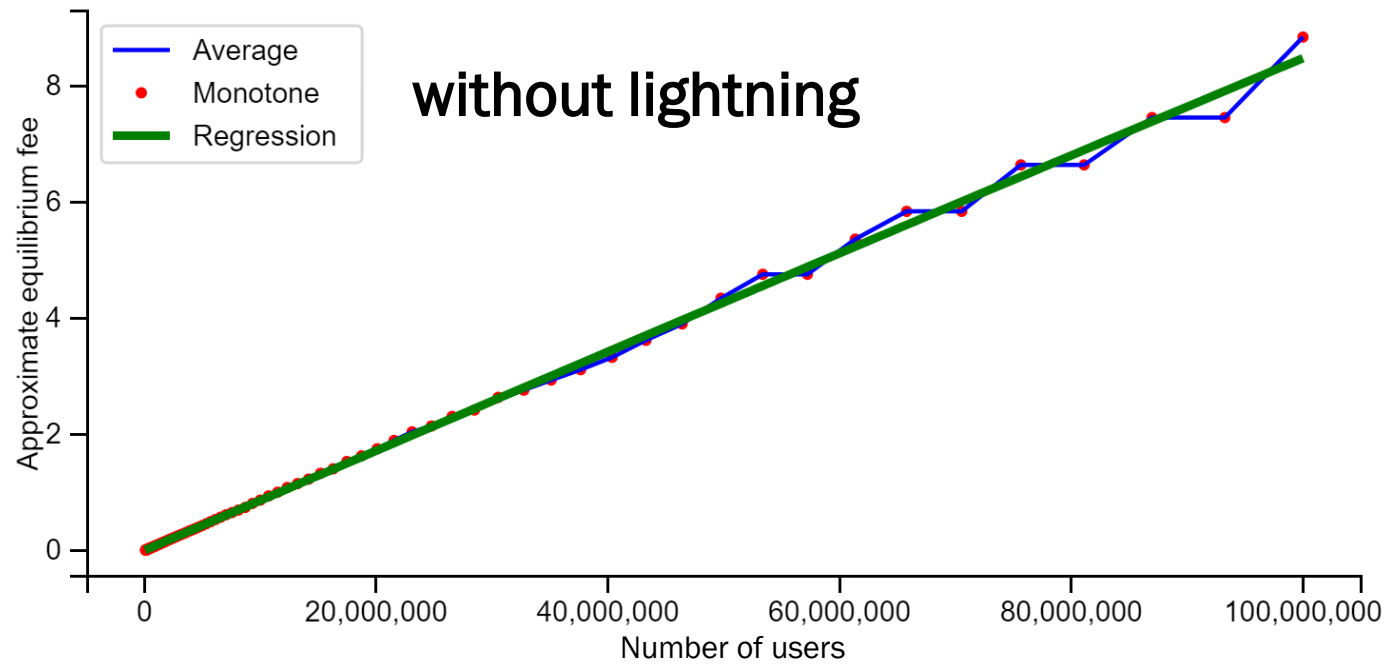
SCALING UP

# Fees

Fees are lower with lightning up to ~20M

Above 20M lightning fees are higher

Miner revenue is proportional to fees (fixed supply)

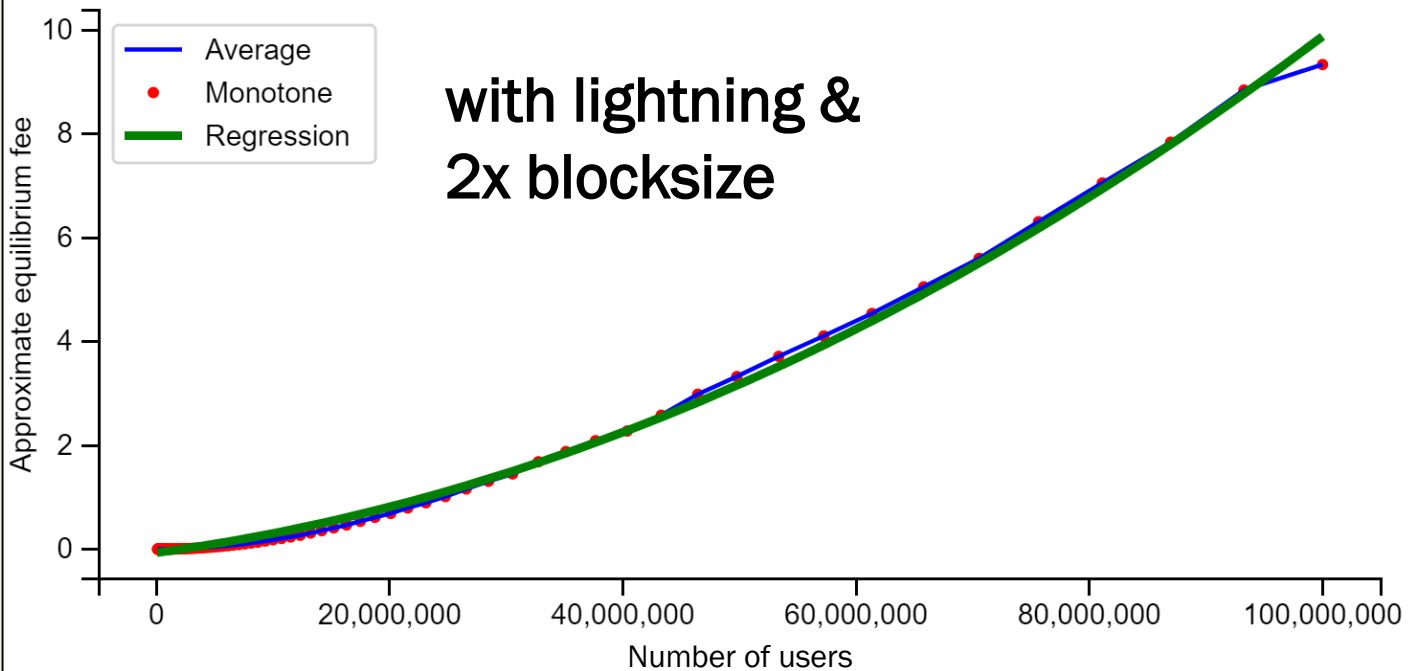
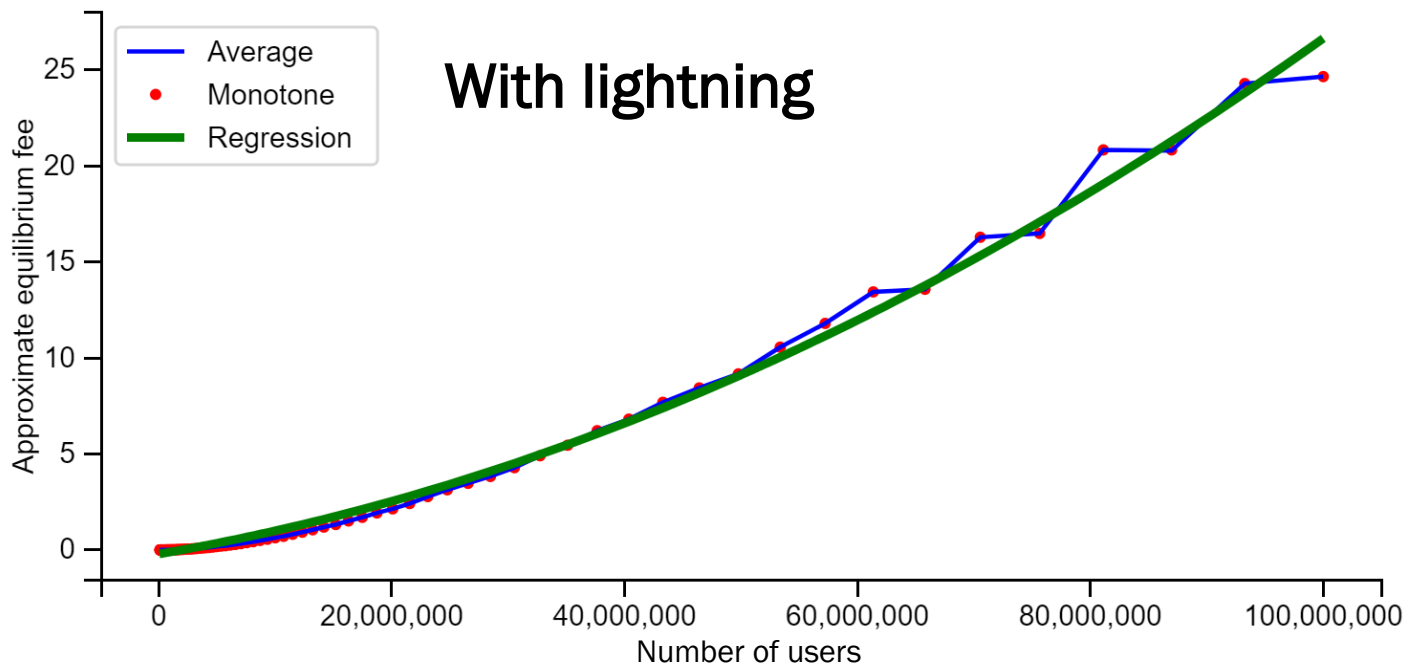


# Fees

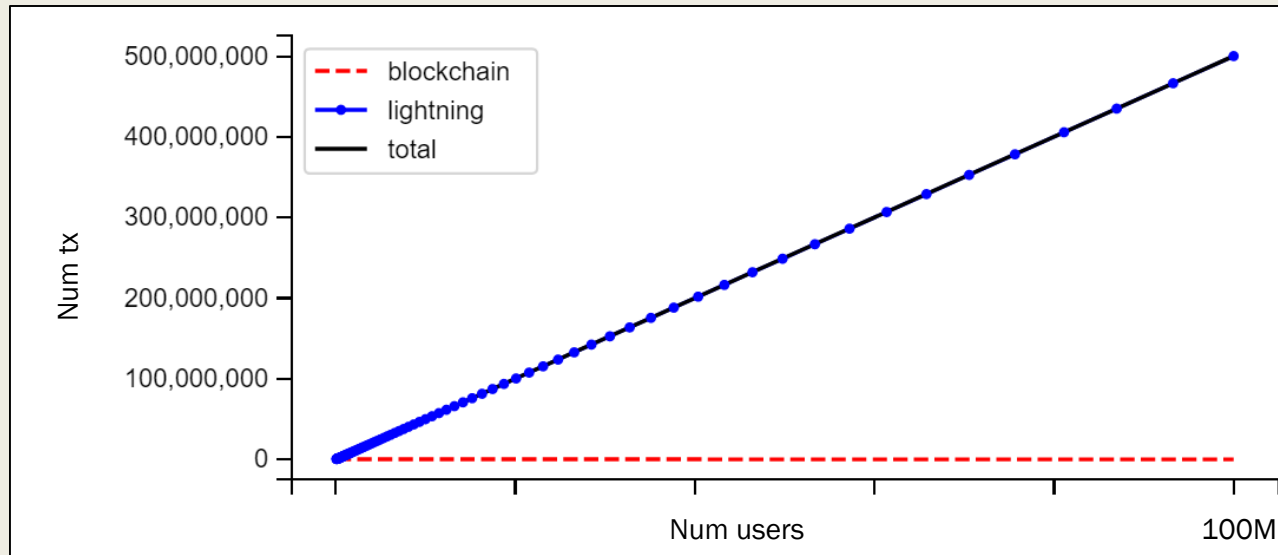
Blochchain  
Fees decline  
more than x2

Supply grew x2

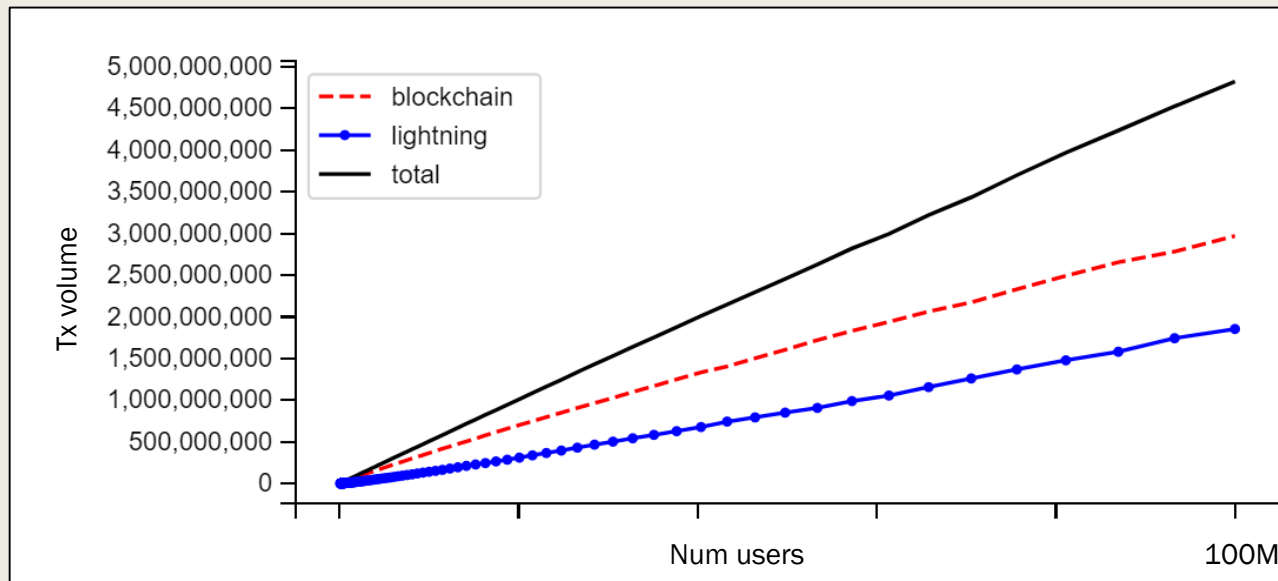
Miner Revenue  
decreased



# Nearly all transactions go through lightning



# But, More volume goes directly via the blockchain



# Conclusion

- Lightning helps (by a lot? Not by a lot?)
- 2X Block increases help (not by a lot)

# What I did not show

- Heterogenous populations
- More complex patterns of flow
- Other transfer distributions

# Concerns

- How fragile / viable is the lightning model?
- Will changes in interest rates & fees have high impact?