

# An Analysis of Bitcoin Exchange Rates

Jacob Smith\*  
University of Houston

March 31, 2015

## Abstract

Bitcoins are digital gold. They are a purely electronic commodity traded for speculative purposes as well as in exchange for goods and services. Just like physical gold, the relative price of bitcoins denominated in different currencies implies a nominal exchange rate. This is a departure from previous literature which treats bitcoin prices themselves as nominal exchange rates. I argue that treating prices as exchange rates is inappropriate as one would not consider the price of physical gold to be an exchange rate. Therefore, this paper characterizes the behavior of nominal exchange rates *implied* by relative bitcoin prices. I show that the implied nominal exchange rate is highly cointegrated with the nominal exchange rate determined in conventional foreign currency exchange markets. I also show that the direction of causality flows from the conventional markets to the bitcoin market and not vice-versa which can explain much of the volatility in bitcoin prices.

**Keywords:** Bitcoin, Cointegration, Cryptocurrency, Exchange Rates

**JEL Classification:** E42, E47, C3

---

\*Special thanks for the helpful comments from David Papell, Dietrich Vollrath, German Cubas, and Christian Murray as well as feedback from attendees of the UH Graduate Research Seminar and Hashers United International Bitcoin Conference. Please address correspondence to: Jacob Smith, Department of Economics, University of Houston, McElhinney 248, Houston, TX 77204-5019. Email: Jacob.BL.Smith@gmail.com. Copyright: Jacob B. Smith ©2014

# 1 Introduction

Bitcoins are digital gold, both in design and behavior. This paper will highlight the similarities between the traditional commodity and its new digital counterpart. In doing so it is evident that the bitcoin market is more mature than previously thought.

The contribution of this paper is to characterize the behavior of bitcoin-implied exchange rates, i.e. the relative price of bitcoin denominated in different currencies. Whereas previous authors have concerned themselves with top-level bitcoin prices, my analysis goes further in studying *relative* bitcoin prices. I argue that the study of these relative prices is a more appropriate endeavor when attempting to compare the behavior of bitcoin prices to conventional currencies. In the same way that one would not call the price of physical gold an exchange rate, neither do I call the price of bitcoins an exchange rate.

Three nominal exchange rates are considered for this analysis: The dollar-euro rate, the dollar-pound rate, and the dollar-Australian dollar rate. These three rates were chosen based on their economic importance and the availability of the data. The data are daily and span the period from September 1<sup>st</sup>, 2011 to January 31<sup>st</sup>, 2014. While bitcoin prices themselves are highly volatile and only weakly correlated with conventional nominal exchange rates, the exchange rates implied by relative bitcoin prices are less volatile and are highly cointegrated with conventional nominal exchange rates.

I employ Vector Error Correction modeling to describe the relationship between implied and market exchange rates. For each exchange rate I estimate one cointegrating vector and find that shocks to the conventional foreign exchange market have significant and permanent effects on implied exchange rates in the bitcoin market. However, shocks to bitcoin-implied exchange rates do not have any noticeable effects on conventional market rates. This one-way causality implies that bitcoin prices must do all of the adjusting in order for implied and market rates to return to parity. In this way, conventional exchange rate volatility drives much of the bitcoin price volatility.

Previous comparisons of bitcoin prices to conventional exchange rates have shown little

similarity. This has led some to conclude that “bitcoin is not a real currency” Yermack (2013). However, in thinking of bitcoin as digital gold the similarities become apparent. I extend my analysis to include gold-implied exchange rates (i.e. relative gold prices) and find that gold and bitcoin behave almost identically. Gold-implied exchange rates are highly cointegrated with market exchange rates and shocks to the conventional market have large persistent effects on the gold market but not vice versa. The similarities between gold and bitcoin speak to the idea that bitcoin is not a currency in the sense that dollars and pounds are currencies, but are much more akin to physical commodities like gold, coffee, and oil.

The rest of the paper proceeds as follows: Section 2 provides a primer on the technology of bitcoins and brief review of the literature. Section 3 presents the data and describes some basic characteristics of bitcoin prices and exchange rates. Section 4 fits an in-sample model and tests the predictability of bitcoin nominal exchange rates. Section 5 concludes.

## 2 What is a Bitcoin?

The recent surge in the popularity of bitcoins has garnered the attention of business professionals and academics unlike any other cryptocurrency. However, most of the discussion on bitcoins has been confined to the computer science literature or blogosphere with almost no academic research devoted to the economics of bitcoin.

The first question one might ask is “What is a Bitcoin?” Ron and Shamir (2012) provide a concise definition:

“Bitcoins are digital coins which are not issued by any government, bank, or organization and rely on cryptographic protocols and a distributed network of users to mint, store, and transfer.”

In other words, bitcoins are a purely digital, highly-liquid asset that are traded for speculative purposes and used as a currency. They are not backed by any tangible asset nor are they sanctioned as legal tender. Thus, bitcoins are non-commodity and non-fiat.

The technology behind bitcoins was first introduced by the pseudonymous Satoshi Nakamoto

in 2009. Fundamentally, the bitcoin network is centered around a massive but decentralized public ledger called the “Block Chain.” This ledger records every bitcoin transaction that’s ever occurred since the network’s genesis. Each bitcoin is given a unique serial number. The Block Chain publicly displays which bitcoins are associated with which accounts. When a transaction occurs the Block Chain is appended to show that a particular bitcoin has moved from one account to another. In this way, the bitcoin market is not unlike the stone wheels of Yap.

The novelty of bitcoin is in the way in which transactions are processed. Suppose Jake wishes to send a bitcoin to Josie. Jake announces his intent to the entire network. The proposed transaction spreads through the network until it finds a “Miner.” A Miner is someone who verifies transactions. The Miner first verifies that Jake has the right to send that particular bitcoin by checking Jake’s “Private Key” which is analogous to an ATM PIN. This lets the Miner know that Jake is the rightful owner of the account. Once Jake’s private key is verified the Miner sets about verifying the authenticity of the particular bitcoin that Jake is trying to send. The Miner does this through a difficult “proof-of-work” algorithm which scrolls through the Block Chain to check the bitcoin’s history. Proof-of-work algorithms are hard to complete but easy to check. Any tampering with the bitcoin’s code or any attempt to spend the same bitcoin twice will generate large deviations from the algorithm’s expected result. For example, I know  $2+2=4$  but if I get sneaky and try to add  $2.0000001+2$  the proof-work-algorithm will report an answer something like 1,729. If the algorithm generates the correct result (or “hash”) then the transaction is deemed to be valid. The Miner then bundles this transaction along with other verified transactions into a “Block.” This new Block is broadcast to the network and appended to the Block Chain. At this point the bitcoin that was in Jake’s account is now publicly recognized to be in Josie’s account and the transaction is complete. The entire process takes approximately ten minutes to complete.

Verifying these transactions is quite costly, requiring specialized equipment, technical

expertise, and a significant amount of electricity. To compensate Miners for their efforts they are awarded newly minted bitcoins every time they complete a Block. It is important to note that the bitcoin supply is not governed by a central bank or any type of monetary policy rule but by the efforts of Miners. This is by design and intended to make the supply of bitcoin similar (at least in theory) to the supply of gold.

As time goes on, the reward for verifying transactions is automatically reduced until the maximum number of 21 million bitcoins have been mined. From then on Miners will no longer be rewarded by the system for processing transactions. At its current pace, the network is expected to mine its last bitcoin in the year 2140. This hard limit on the total supply of bitcoins is feared to put deflationary pressure on prices and to thus encourage hoarding. Indeed, Ron and Shamir (2012) find that 78% of all bitcoins are held in accounts which receive coins but never spend them. Questions about the long-run incentives in the bitcoin market lead Kroll, Davey, and Felten (2013) to conclude that some form of governance will be required for long-run sustainability of the market.

Others have also leveled criticisms against the economic viability of bitcoin. Meiklejohn et al. (2013) show that interactions in the bitcoin market aren't that anonymous as the Block Chain provides a public record of all transactions. This is an issue for bitcoin as the anonymity of illicit transactions was and is one of the major draws for its utility as a currency (Christin 2012). Pirrong (2013) attacks the efficiency of the market arguing that too few actors and small market cap leave the market vulnerable to manipulation. Even more concerning, Eyal and Sirer (2014) describe a strategy in which bitcoin Miners may unilaterally cheat the system to receive more than their fair share of mining proceeds.

Yermack (2013) questions the maturity of the market by asking "Is Bitcoin a Real Currency?" The author shows that there is almost no correlation between the price of bitcoins and conventional exchange rates. Moreover, the volatility of bitcoin prices is an order of magnitude higher than the volatility observed in conventional nominal exchange rates. This leads Yermack to conclude that "bitcoin behaves more like a speculative investment than a

currency.”

This paper looks deeper into the behavior of the bitcoin market and endeavors to explain the extraordinarily high price volatility. Understanding the technology and construction of bitcoins is crucial to understanding the behavior of agents in the bitcoin market. Like other commodities (such as gold), bitcoins are bought and sold primarily for investment purposes and assume a secondary role as a currency. And like these other commodities, the relative price of bitcoins denominated in different currencies implies an exchange rate. That is, the dollar price of bitcoins ( $\$/\text{₤}$ ) divided by the Euro price of bitcoins ( $\text{€}/\text{₤}$ ) implies a dollar-euro exchange rate ( $\$/\text{€}$ ). Focusing on this *bitcoin-implied exchange rate* gives insight into the behavior of the bitcoin market. Contrasting the behavior of bitcoin-implied exchange rates with similar gold-implied exchange rates allows a more direct comparison between traditional commodity money and the burgeoning market for cryptocurrency. I find that bitcoins are subject to the same motivations and dynamics as physical gold which leads me to conclude that the bitcoin market, though small, is relatively mature.

### 3 Data

Daily data on bitcoin prices were downloaded from bitcoincharts.com. The prices are those which prevailed on the Mt. Gox<sup>1</sup> exchange at 12:00 a.m. GMT (UTC). There are any number of bitcoin exchanges with freely floating prices, however, Mt. Gox was by far the largest and most active (Meiklejohn et al. 2013). Data on bitcoin prices denominated in US Dollars, British Pounds, Euros, and Australian Dollars were collected for the period September 1<sup>st</sup>, 2011 to January 31<sup>st</sup>, 2014<sup>2</sup>. Implied exchange rates were then calculated by dividing the Dollar price of bitcoins ( $\$/\text{₤}$ ) by the foreign price of bitcoins ( $\text{₤}/\text{₤}$ ,  $\text{€}/\text{₤}$ ,

---

<sup>1</sup>Not only was Mt. Gox the largest and most active bitcoin exchange, it was also one of the most active bitcoin market places in general (Meiklejohn et al. 2013).

<sup>2</sup>Data are available after January 31<sup>st</sup>, 2014, however, in early February 2014 news broke that Mt. Gox’s security protocols had been compromised allowing hackers to seize bitcoins stored in customers’ accounts. Thus, there was a subsequent run on Mt. Gox bitcoins which caused the price to plummet and forced Mt. Gox into bankruptcy.

$A\$/\text{€}$ ). Conventional market exchange rates as well as gold prices were obtained from the St. Louis Fed's FRED database. The nominal exchange rates are the "noon buying rates in New York City for cable transfers payable in foreign currencies." While gold prices are 3:00 p.m. (London Time) fixing prices which prevailed in the London Bullion Market. Figures 1-5 illustrate the data and Table 1 provides some descriptive statistics.

The descriptive statistics in Table 1 reiterate the findings of Yermack (2013): bitcoin prices are indeed extraordinarily volatile when compared with conventional market exchange rates. A fact illustrated in Figure 1. However, if one considers the bitcoin implied exchange rates one notices that the mean and standard deviation are almost identical to the market exchange rates. This fact is born out in Figures 3-5. While it is true that bitcoin implied rates remain slightly more volatile than conventional market rates, it is clear the the implied and market rates follow each other very closely. This is despite the fact that bitcoin prices themselves are almost totally uncorrelated with market exchange rates.

One sees a similar pattern with physical gold. Gold prices themselves are highly volatile and seemingly uncorrelated with market exchange rates, however, gold-implied exchange rates track closely with market rates.

The relationship between the implied and market exchange rate is motivated by arbitrage. When the bitcoin (or gold) implied exchange rate finds itself out of parity with the conventional market rate there exists an opportunity for arbitrage. As agents take advantage of these arbitrage opportunities the exchange rates are driven back together. For example, suppose the implied dollar-euro rate was greater in the bitcoin market than in the conventional market. This means the euro is relatively stronger in the bitcoin market than in the conventional market. Thus, a trader could exchange dollars for euros in the conventional market (where euros are relatively cheap) and take those euros to buy bitcoins in the bitcoin market (where euros are relatively valuable). The purchase of bitcoins with euros drives up the euro price of bitcoin thereby driving down the implied exchange rate.

Such arbitrage is common whenever commodities are traded in different currencies (Sjaas-

tad and Scacciavillani 1996). These arbitrage trades often result in the series being cointegrated. Two non-stationary series are said to be cointegrated if a linear combination of the series is stationary. If one considers the spread between implied and market rates (i.e.  $\$/\text{€}_{market} - \$/\text{€}_{implied}$ ) in Figure 6 it seems clear that the linear combination of the series is stationary about 0. This is mirrored in Figures 7 and 8 for the dollar-pound and dollar-Australian dollar rates respectively.

One interesting comparison that can be made between the gold and bitcoin implied rates is the relative size of the so-called arbitrage bands. The bands are much larger for the bitcoin implied rates than for the gold implied rates. The width of the arbitrage band is dictated by transaction costs, i.e. arbitrage trades are only made once the profits exceed the transactions costs. Thus, higher transaction costs allow for higher disparity. Because the arbitrage bands are much larger for bitcoins than for gold it must be that transactions costs are higher in the bitcoin market than in the gold market. The exact reasoning for this is outside the scope of this paper but further research into the size and dynamics of the exchange rate spread could prove an interesting topic for further discussion.

## 4 Empirical Analysis

Engle and Granger (1987) show that if two series are cointegrated then a simple regression of  $\Delta y_t$  on  $\Delta x_t$  is misspecified. A system of  $I(1)$  unit root processes is said to be cointegrated if there exists some linear combination of the series which is stationary, or  $I(0)$ . If we let  $\mathbf{y}_t$  be a vector of time series then the system is cointegrated if there exists some non-zero vector  $\boldsymbol{\beta}$  such that  $\boldsymbol{\beta}'\mathbf{y}_t$  is stationary. The system is said to be in “equilibrium” when  $\boldsymbol{\beta}'\mathbf{y}_t = \mathbf{0}$  and out of equilibrium when  $\boldsymbol{\beta}'\mathbf{y}_t \neq 0$ . We can denote this “equilibrium error” as  $\mathbf{z}_t = \boldsymbol{\beta}'\mathbf{y}_t$ .



Consider the bivariate system,

$$y_t + \alpha x_t = \epsilon_t, \quad \epsilon_t = \epsilon_{t-1} + \xi_t \quad (1)$$

$$y_t + \beta x_t = \nu_t, \quad \nu_t = \rho \nu_{t-1} + \zeta_t, \quad |\rho| < 1 \quad (2)$$

where  $\xi_t$  and  $\zeta_t$  are white noise. Note that the reduced form of  $y_t$  and  $x_t$  will be functions of both  $\epsilon$  and  $\nu$ . Because  $\epsilon_t$  is  $I(1)$  it must be the case that  $y_t$  and  $x_t$  are also  $I(1)$ . Now consider (2) which is a linear combination of  $y_t$  and  $x_t$ . Because  $\nu_t$  is stationary it must be the case that  $y_t + \beta x_t$  is also stationary. Therefore,  $y_t$  and  $x_t$  are cointegrated with a vector  $\beta' = (1, \beta)$ .

Engle and Granger (1987) show that we can rewrite the system as,

$$\Delta y_t = \alpha \delta z_{t-1} + \eta_{1t} \quad (3)$$

$$\Delta x_t = -\delta z_{t-1} + \eta_{2t} \quad (4)$$

where  $\delta = (1 - \rho)/(\beta - \alpha)$  and the  $\eta$ 's are linear combinations of  $\epsilon_t$  and  $\nu_t$ . Recall that  $z_t = y_t + \beta x_t$ . Thus, equations (3) and (4) illustrate the vector error correction (VEC) representation of the system which describes how the series respond to disequilibrium.

If, however,  $\mathbf{y}_t$  is a VAR( $p$ ) the VEC representation can be expanded and written as,

$$\Delta \mathbf{y}_t = \alpha \beta' \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \epsilon_t$$

For this analysis,  $\Delta \mathbf{y}_t$  is a 2x1 vector  $\begin{bmatrix} \Delta y_t \\ \Delta x_t \end{bmatrix}$  where the  $\Delta y_t$  is the first difference in the natural log of the implied exchange rate and  $\Delta x_t$  is the first difference in the natural log of the market exchange rate. The vectors  $\alpha$  and  $\beta$  are also 2x1 and contain the adjustment parameters and cointegrating vector respectively. A simple VAR of  $\Delta y_t$  and  $\Delta x_t$  would omit  $\alpha \beta' \mathbf{y}_{t-1}$  and therefore be misspecified if indeed the series are cointegrated.

The matrix  $\Gamma_p$  is a 2x2 matrix of lag parameters

$$\mathbf{\Gamma}_p = \begin{bmatrix} \gamma_{yy,p} & \gamma_{yx,p} \\ \gamma_{xy,p} & \gamma_{xx,p} \end{bmatrix}$$

Applying the VEC method to implied and market exchange rates follows Sjaastad and Scacciavillani (1996). In their paper, Sjaastad and Scacciavillani estimate a cointegrating relationship between gold-implied exchange rates and the market rate then attempt to use this vector to forecast the nominal market rate. The authors show that the adoption of floating exchange rate regimes has contributed significantly to the volatility of gold prices. I will show that this is also a major factor in the extreme volatility in bitcoin prices. Sjaastad and Scacciavillani (1996) also find that that the direction of causality flows from the nominal exchange rate market to the gold market, but not vice versa. We will see that the bitcoin market exhibits the same behavior. At the end of this section, I replicate portions of Sjaastad and Scacciavillani (1996) using my data and find similar results.

The contribution of this paper is to apply Sjaastad and Scacciavillani’s methodology to the bitcoin market. Doing so will shed light on the dynamics of the market as well as illustrate the similarities between purely digital bitcoins and other physical commodities.

Determining the order of the VAR means considering five lag-selection methodologies. The five methodologies all involve minimizing some criterion, either the Likelihood-Ratio, the Final Prediction Error (FPE), the Akaike Information Criterion (AIC), the Hannan-Quinn Information Criterion (HQIC), or the Schwartz Bayesian Information Criterion (SBIC). The lag length which minimizes the majority of these five criteria is taken to be the most appropriate. For the dollar-euro and the dollar-Australian dollar rates a lag length of 2 is selected. For the dollar-pound rates I use a lag-length of 5.

From there I apply Johansen’s (1988, 1991, 1995) methodology to formally tests for the existence of cointegration. The results of the Johansen tests show that for all three currencies I strongly reject the null of no cointegrating vector but fail to reject the null of at most 1 cointegrating vector. The results of these tests can be found in Table 2.

The Johansen tests confirm the presence of cointegration for all three currencies making VEC modeling the most appropriate technique. Table 3 reports the parameter estimates of (5) when considering bitcoin implied rates and conventional market rates.

In examining to the results from Panel 3a we see that the estimates of  $\alpha_1$  are negative and highly significant for each of the exchange rates. Recall that  $\alpha_1$  is the way in which the first series responds to disequilibrium, i.e. the implied rate's "error correction." The fact that  $\alpha_1$  is negative and significant for all three currencies indicates that when the bitcoin-implied exchange rate finds itself out of equilibrium with the conventional market exchange rate, the implied rate quickly adjusts to restore balance. Moreover, this error correction dynamic explains between 36% and 49% of the movement in the implied rates meaning that shocks to market exchange rates have enormous knock-on effects in the bitcoin market. However, if one considers the output in Panel 3b one sees that the same error correction dynamic is not present when the market rate is the response variable. The estimated coefficients on  $\alpha_2$  are insignificant for each of the currencies, thus, the conventional market does not respond to disequilibrium with the bitcoin market.

A further illustration of this one-way causality can be found in the plots of the impulse-response functions. Figures 9-11 show the response of the implied rate to a shock in the market rate for the  $\$/\text{€}$ ,  $\$/\text{£}$ , and  $\$/\text{A\$}$  respectively. One can see that shocks to the implied rate are significant and persistent, i.e. shocks to the market rate have positive effects on the implied rate which never wear off. For all three currencies, the shocks are almost fully realized within the first few days indicating an active and alert pool of traders.

However, if one considers the IRF's in Figures 12-14, shocks to the implied rate yield almost no effects on the conventional market rate. In other words, the conventional market does not react to changes in the bitcoin market. This is intuitive as the total capitalization of the entire bitcoin market was less than \$5 billion at the time of this writing whereas M2 was nearly \$11.5 trillion. The bitcoin market is simply not large enough to move the global currency markets but global currency markets are more than large enough to move

the bitcoin market.

The parameters  $\beta$  define the cointegrating vector. Following Johansen (1988, 1991, 1995)  $\beta_1$  is normalized to 1 for identification. In the bivariate system, this means  $\beta_2$  defines the equilibrium condition. Recall that equilibrium occurs where  $0 = y_t + \beta_2 x_t$ , or when  $y_t = -\beta_2 x_t$ . Thus,  $\hat{\beta}_2$  gives an estimate of the relationship between  $y_t$  and  $x_t$  in equilibrium.

In Panel 3b we see that the estimated  $\beta_2$  is very precise but not statistically different from -1 for the  $\$/\text{€}$  rate. Therefore, we fail to reject the null that the equilibrium condition is for parity between the implied and market rates.  $y_t = x_t$ . The  $\beta_2$  estimates for the  $\$/\text{£}$  and  $\$/\text{A\$}$  are also very precise and close to -1, however, they do remain statistically different from -1. Thus, I cannot make the same claim that the equilibrium condition is parity. That being said, the absolute difference from -1 is small in both cases.

Taken together, this tells us that when the bitcoin market is out of equilibrium with the conventional exchange market bitcoin prices are forced to make the entire adjustment back to parity. This is the same source of volatility described by Sjaastad and Scacciavillani (1996) within the gold market. To fully illustrate the similarities between bitcoins and physical gold I replicate the above analysis using gold-implied exchange rates rather than bitcoin-implied exchange rates.

As with bitcoin-implied rates, Johansen tests for cointegration in Table 4 indicate the presence of one cointegrating vector for both the  $\$/\text{€}$  and  $\$/\text{£}$  rate. Table 5 presents the estimates from the VEC model. The estimates on  $\alpha_1$  are negative and highly significant for both currencies in Panel 5a where the gold-implied exchange rate is the response variable. Once again, this indicates that as the gold-implied rate finds itself out of equilibrium with the market rate the implied rate adjusts to restore parity. This error-correction dynamic accounts for 14% and 8% of the variation in the implied  $\$/\text{€}$  and  $\$/\text{£}$  rates respectively. While market exchange rate volatility is a contributing factor to gold price volatility it is not as significant a factor as in the bitcoin market. Moreover, in considering the magnitudes of  $\hat{\alpha}_1$  we see that the gold-implied rate adjusts much more rapidly than do bitcoin-implied rate.

This gives us some perspective as to the relative size and maturity of the bitcoin market.

In turning to Panel 5b there is little evidence to suggest that the market exchange rates respond to disparities with the implied rate. The estimate of  $\alpha_2$  for the  $\$/\mathcal{L}$  rate is not statistically different from 0 while the estimate for the  $\$/\mathcal{E}$  rate is only marginally significant. This is the same one-way, market-to-implied causality that we saw in the bitcoin analysis. Again, this one-way causality is born out in the graphs of the impulse-response functions. Figures 15-18 show the permanent effects of a market exchange rate shock to the gold-implied exchange rate while illustrating the negligible, transitory effects of a gold-implied rate shock to the currency market.

As in the bitcoin analysis,  $\beta_1$  is normalized to 1 for identification while  $\beta_2$  describes the equilibrium condition. We see that for both the  $\$/\mathcal{E}$  and  $\$/\mathcal{L}$  rates  $\hat{\beta}_2$  is exactly -1 meaning the equilibrium condition is for the implied rate to exactly equal the market rate. Again, the fact that  $\hat{\beta}_2$  is precisely -1 in Panel 5b while somewhat less precise in Panel 5a may be indicative of a smaller, more immature bitcoin market. That is, traders in the bitcoin market might not be as sophisticated or motivated to act on arbitrage opportunities. It could also simply be a function of the presumably larger transactions costs in the bitcoin market which prevent arbitrage trades from occurring and therefore allow for larger disparities between the implied and market rates.

## 5 Conclusion

I show that the most appropriate way to think about bitcoins is as digital gold. While nominal bitcoin prices are extremely volatile and seemingly uncorrelated with other nominal exchange rates, relative bitcoin prices or *implied* nominal exchange rates are indeed highly cointegrated with conventional market exchange rates. This mirrors the relationship between physical gold and conventional nominal exchange rates. Driven by arbitrage opportunities, relative bitcoin prices adjust rapidly to restore parity with market exchange rates, however

market exchange rates seem unaffected by changes in bitcoin prices. Indeed, almost half of the movement in relative bitcoin prices can be explained by this cointegrating vector. Thus, floating nominal exchange rates are a major source of price volatility in the bitcoin market just as they are in conventional commodity markets. While the bitcoin market remains relatively small and highly volatile it appears as though there is a deep and active pool of traders which maintain a level of market efficiency. However, floating exchange rates can only explain part of bitcoin's volatility. It is clear that the market is still small enough to be manipulated by a small handful of investors and questions are being raised about the robustness of bitcoin's underlying technology. It remains to be seen whether bitcoin's innovations as a currency can withstand these pressures.

## References

- [1] Nicolas Christin. “Traveling the Silk Road: A Measurement Analysis of a Large Anonymous Online Marketplace”. Unpublished. 2012. URL: [https://www.cylab.cmu.edu/files/pdfs/tech\\_reports/CMUCyLab12018.pdf](https://www.cylab.cmu.edu/files/pdfs/tech_reports/CMUCyLab12018.pdf).
- [2] Robert F Engle and Clive W J Granger. “Co-integration and Error Correction: Representation, Estimation, and Testing”. *Econometrica* 55.2 (1987), pp. 251–76.
- [3] Ittay Eyal and Emin Gün Sirer. “Majority is not Enough: Bitcoin Mining is Vulnerable”. Unpublished. 2014. URL: <http://www.cs.cornell.edu/~ie53/publications/btcProcArXiv.pdf>.
- [4] Soren Johansen. “Statistical Analysis of Cointegration Vectors”. *Journal of Economic Dynamics and Control* 12.2-3 (1988), pp. 231–254.
- [5] Soren Johansen. “Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models”. *Econometrica* 59.6 (1991), pp. 1551–80.
- [6] Soren Johansen. “A Statistical Analysis for Cointegration for I(2) Variables”. *Econometric Theory* 11.1 (1995), pp. 25–59.
- [7] Lutz Kilian. “Exchange Rates and Monetary Fundamentals: What Do We Learn from Long-Horizon Regressions?” *Journal of Applied Econometrics* 14.5 (1999), pp. 491–510.
- [8] Joshua Kroll, Ian Davey, and Edward Felten. “The Economics of Bitcoin Mining, or Bitcoin in the Presence of Adversaries”. Unpublished. 2013. URL: [https://www.cs.princeton.edu/~kroll/papers/weis13\\_bitcoin.pdf](https://www.cs.princeton.edu/~kroll/papers/weis13_bitcoin.pdf).
- [9] Sarah Meiklejohn et al. “A Fistful of Bitcoins: Characterizing Payments Among Men with No Names”. Unpublished. 2013. URL: <https://cseweb.ucsd.edu/~smeiklejohn/files/login13.pdf>.
- [10] Tyler Moore and Nicolas Christin. “Beware the Middleman: Empirical Analysis of Bitcoin-Exchange Risk”. Unpublished. 2013. URL: <http://fc13.ifca.ai/proc/1-2.pdf>.
- [11] Satoshi Nakamoto. “Bitcoin: A Peer-to-Peer Electronic Cash System”. Unpublished. 2009. URL: <https://bitcoin.org/bitcoin.pdf>.
- [12] Craig Pirrong. “Was the Bitcoin Flock Just Sheared?” *Streetwise Professor Blog* (2013). URL: <http://streetwiseprofessor.com/index.php?s=bitcoin>.
- [13] Dorit Ron and Adi Shamir. “Quantitative Analysis of the Full Bitcoin Transaction Graph”. Unpublished. 2012. URL: <https://eprint.iacr.org/2012/584.pdf>.
- [14] Larry A Sjaastad and Fabio Scacciavillani. “The Price of Gold and the Exchange Rate”. *Journal of International Money and Finance* 15.6 (1996), pp. 879–897.
- [15] *VEC Intro*. 13th ed. Stata Corp. 2014. URL: <http://www.stata.com/manuals13/tsvecintro.pdf>.
- [16] David Yermack. “Is Bitcoin a Real Currency?” NBER Working Paper Series Number 19747. 2013. URL: <http://www.nber.org/papers/w19747>.

Table 1: Descriptive Statistics

	Bitcoin				Gold										
	Bitcoin Prices		Implied Rates		Gold Prices		Implied Rates		Market Rates						
	\$/B	€/B	£/B	A\$/B	\$/€	\$/£	\$/A\$	€/oz	£/oz	\$/€	\$/£	\$/A\$			
Mean	118.41	87.31	80.86	145.53	1.31	1.57	0.99	1,541.31	1,156.61	972.54	1.34	1.59	1.32	1.58	1.00
Std. Dev	245.86	178.67	156.16	289.01	0.05	0.05	0.05	170.81	143.04	109.06	0.06	0.04	0.04	0.04	0.06
High	1,230.00	885.10	738.00	1,333.64	1.58	1.87	1.21	1,895.00	1,382.27	1,182.82	1.49	1.67	1.43	1.66	1.08
Low	2.05	1.54	1.40	2.07	1.19	1.45	0.87	1,192.00	873.14	730.10	1.21	1.49	1.21	1.48	0.87
$N$	884	882	877	859	882	877	859	774	774	774	774	774	605	605	605

Table 2: Johansen Tests for Cointegration

	\$/€		\$/£		\$/A\$	
	Log Likelihood	Trace Statistic	Log Likelihood	Trace Statistic	Log Likelihood	Trace Statistic
Maximum Rank	4003.69	158.47	4003.69	158.47	4003.69	158.47
0	4003.69	158.47	3977.25	36.45	3484.72	145.44
1	4082.79	0.29*	3995.44	0.07*	3557.19	0.51*
Lags	2		5		2	
$N$	602		598		595	

\* Indicates failure to reject  $H_0$  : max rank  $\leq x$



Table 3a: Vector Error Correction Model

Response Variable: Natural Log of Bitcoin-Implied Exchange Rates

	\$/€			\$/£			\$/A\$		
	Estimate	$R^2$	$N$	Estimate	$R^2$	$N$	Estimate	$R^2$	$N$
$\alpha_1$	-0.500*** (0.037)	0.36	602	-0.345*** (0.056)	0.49	598	-0.631*** (0.049)	0.46	595
$\beta_1$	1 na			1 na			1 na		
$\gamma_{yy,1}$	-0.144*** (0.036)			-0.541*** (0.057)			-0.233*** (0.038)		
$\gamma_{yy,2}$				-0.385*** (0.055)					
$\gamma_{yy,3}$				-0.392*** (0.049)					
$\gamma_{yy,4}$				-0.152*** (0.037)					
$\gamma_{xy,1}$	-0.067 (0.092)			-0.078 (0.149)			0.231* (0.126)		
$\gamma_{xy,2}$				0.201 (0.148)					
$\gamma_{xy,3}$				0.472*** (0.146)					
$\gamma_{xy,4}$				0.369*** (0.144)					

\* Significant at 10% level, \*\* Significant at 5% level, \*\*\* Significant at 1% level.

Table 3b: Vector Error Correction Model

Response Variable: Natural Log of Conventional Market Exchange Rates

	\$/€			\$/£			\$/A\$		
	Estimate	R <sup>2</sup>	N	Estimate	R <sup>2</sup>	N	Estimate	R <sup>2</sup>	N
$\alpha_2$	-0.011 (0.018)	0.01	602	-0.014 (0.016)	0.00	598	-0.017 (0.017)	0.01	595
$\beta_2$	-0.996*** (0.003)			-0.989*** (0.004)			-0.897*** (0.024)		
$\gamma_{yx,1}$	0.011 (0.017)			0.024 (0.017)			-0.017 (0.013)		
$\gamma_{yx,2}$				0.020 (0.016)					
$\gamma_{yx,3}$				0.024* (0.014)					
$\gamma_{yx,4}$				0.028*** (0.011)					
$\gamma_{xx,1}$	-0.047 (0.044)			-0.061 (0.044)			-0.033 (0.044)		
$\gamma_{xx,2}$				-0.016 (0.044)					
$\gamma_{xx,3}$				-0.051 (0.043)					
$\gamma_{xx,4}$				-0.047 (0.043)					

Significance of the Cointegrating Equation

	\$/€	\$/£	\$/A\$
$\chi^2$	81047.29***	57583.73***	1419.54***

\* Significant at 10% level, \*\* Significant at 5% level, \*\*\* Significant at 1% level.

Maximum Rank	\$/€			\$/£		
	Log Likelihood	Trace Statistic	5% CV	Log Likelihood	Trace Statistic	5% CV
0	4003.69	158.47	12.53	5193.23	109.51	12.53
1	4082.79	0.29*	3.84	5247.97	0.03*	3.84
Lags	2			5		
$N$	602			598		

\* Indicates failure to reject  $H_0 : \max \text{rank} \leq x$

Response Variable: Natural Log of Gold-Implied Exchange Rates						
	\$/€			\$/£		
	Estimate	$R^2$	$N$	Estimate	$R^2$	$N$
$\alpha_1$	-1.459*** (0.238)	0.14	582	-0.745*** (0.218)	0.08	582
$\beta_1$	1 na			1 na		
$\gamma_{yy,1}$	0.410** (0.211)			-0.004 (0.193)		
$\gamma_{yy,2}$	0.392** (0.162)			0.181 (0.159)		
$\gamma_{yy,3}$	0.192* (0.109)			0.122 (0.273)		
$\gamma_{xy,1}$	-0.473** (0.211)			-0.053 (0.194)		
$\gamma_{xy,2}$	-0.431*** (0.167)			-0.169 (0.159)		
$\gamma_{xy,3}$	-0.238** (0.116)			-0.173 (0.114)		

\* Significant at 10% level

\*\* Significant at 5% level

\*\*\* Significant at 1% level

Table 5b: Vector Error Correction Model

Response Variable:  
Natural Log of Conventional Market Exchange Rates

	\$/€			\$/£		
	Estimate	$R^2$	$N$	Estimate	$R^2$	$N$
$\alpha_2$	-0.448*	0.02	582	0.078	0.02	582
	(0.254)			(0.218)		
$\beta_2$	-1.000***			-1.000***		
	(0.000)			(0.000)		
$\gamma_{yx,1}$	0.498**			0.214		
	(0.219)			(0.209)		
$\gamma_{yx,2}$	0.528***			0.385**		
	(0.173)			(0.171)		
$\gamma_{yx,3}$	0.233**			0.268**		
	(0.116)			(0.120)		
$\gamma_{xx,1}$	-0.526**			-0.236		
	(0.225)			(0.209)		
$\gamma_{xx,2}$	-0.543***			-0.358**		
	(0.178)			(0.172)		
$\gamma_{xx,3}$	-0.256**			-0.288**		
	(0.124)			(0.123)		

Significance of the Cointegrating Equation

	\$/€	\$/£
$\chi^2$	1.40x10 <sup>7</sup> ***	3.51x10 <sup>7</sup> ***

\* Significant at 10% level

\*\* Significant at 5% level

\*\*\* Significant at 1% level

Figure 1: Bitcoin Prices

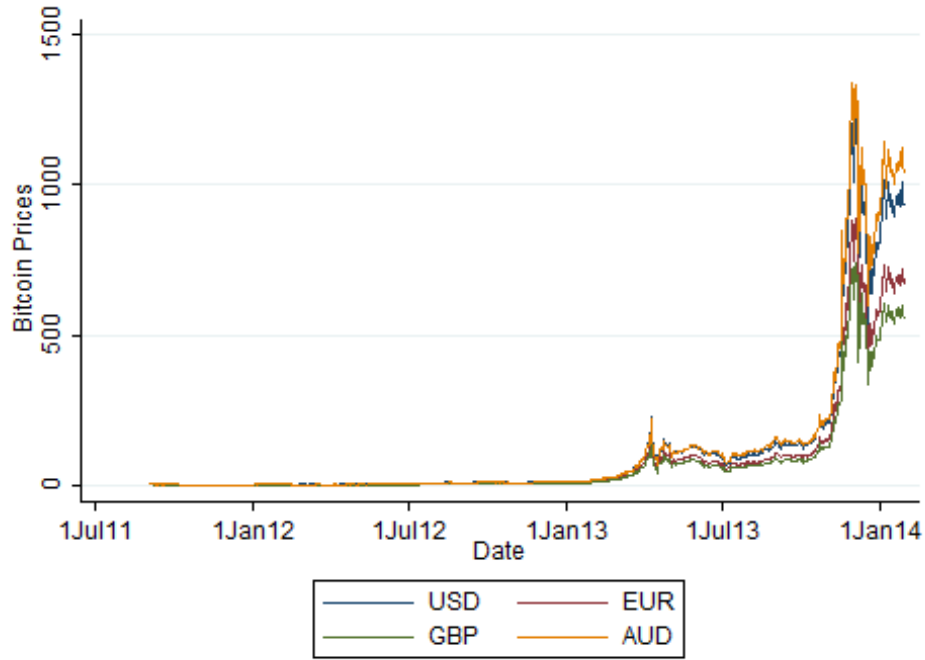


Figure 2: Gold Prices

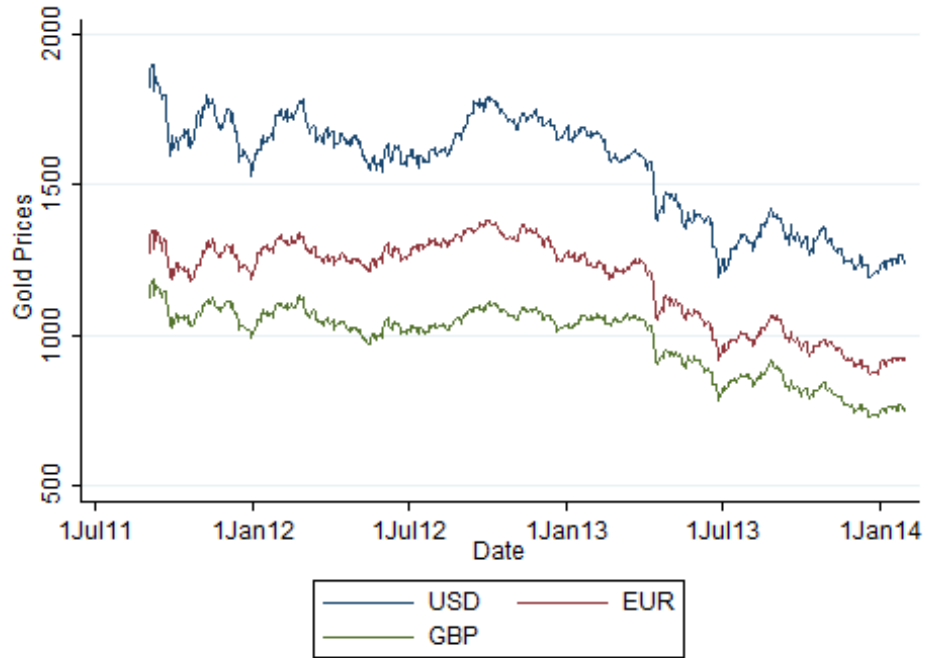


Figure 3: Dollar-Euro Nominal Exchange Rate (\$/€)

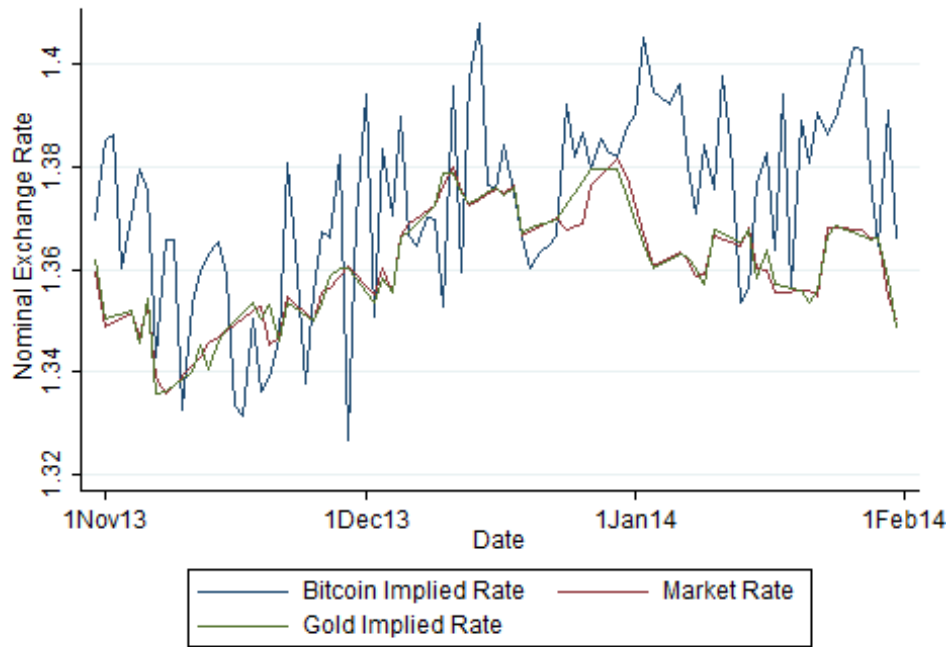


Figure 4: Dollar-Pound Nominal Exchange Rate (\$/£)

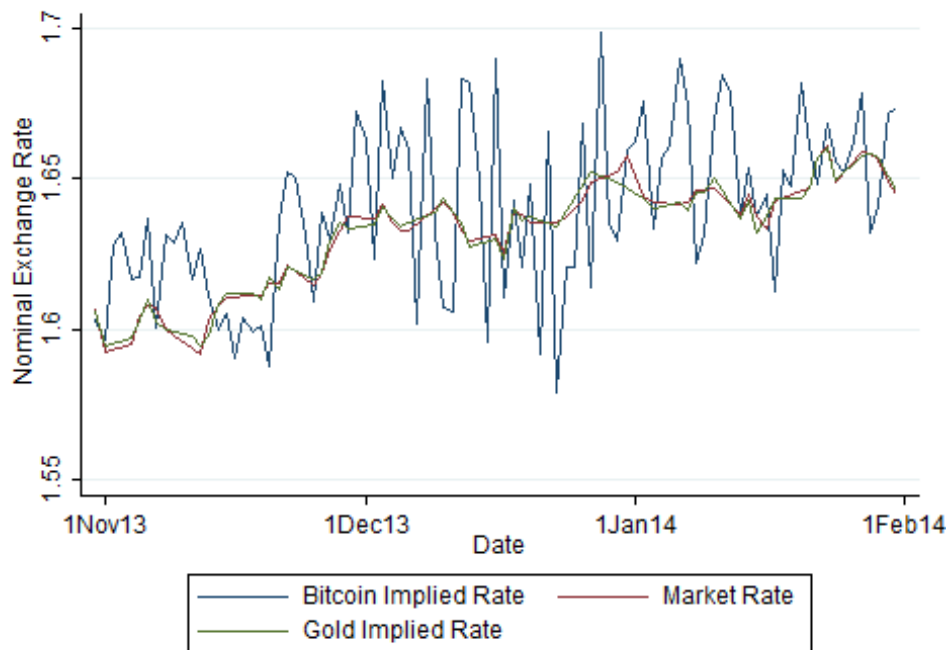


Figure 5: Dollar-Australian Dollar Nominal Exchange Rate ( $\$/A\$\text{)$

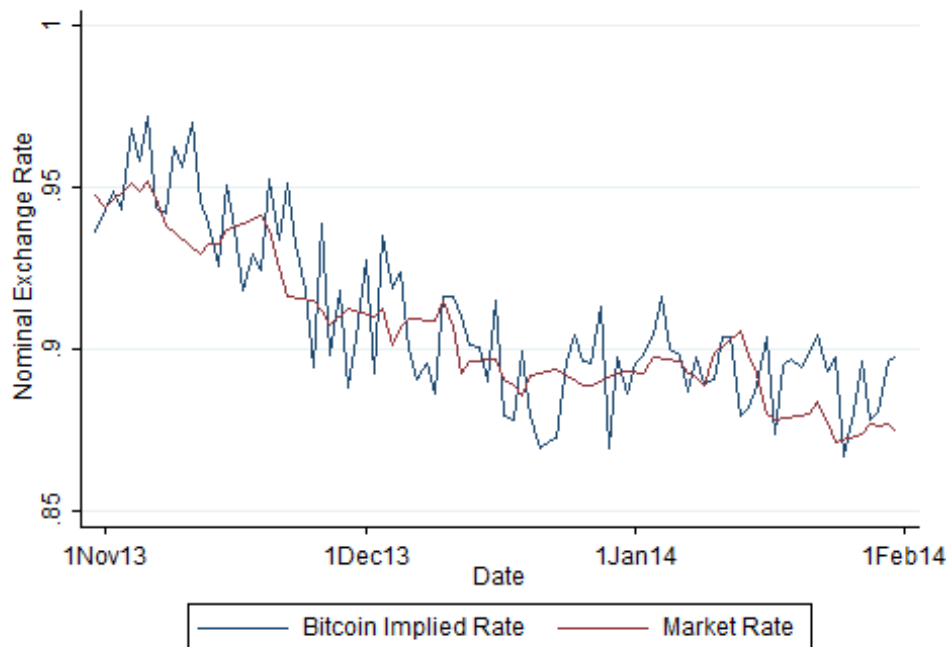


Figure 6: Implied vs. Market Rate Spread for  $\$/\text{€}$  Rate

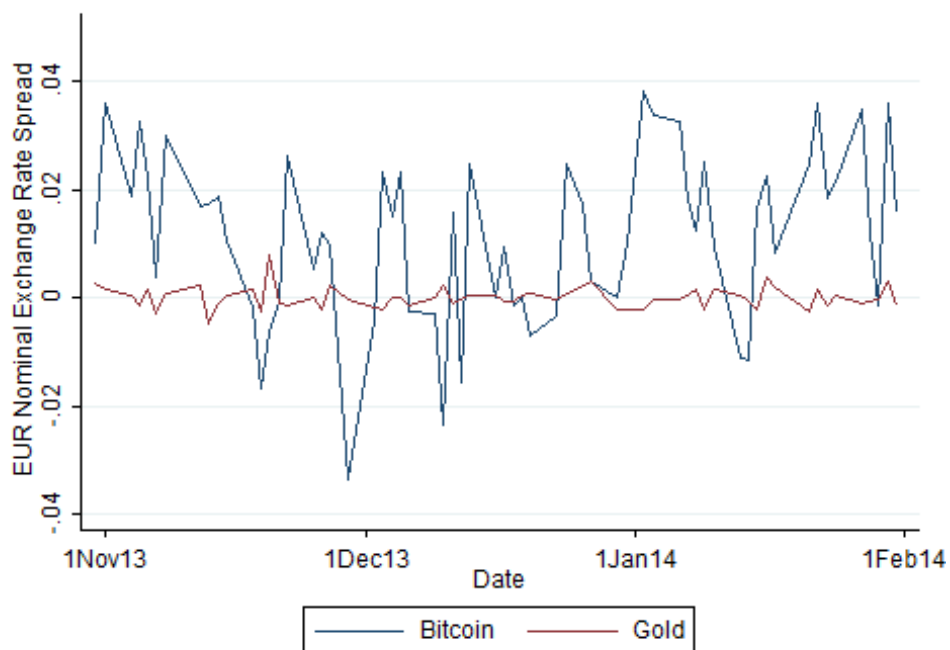


Figure 7: Implied vs. Market Rate Spread for  $\$/\pounds$  Rate

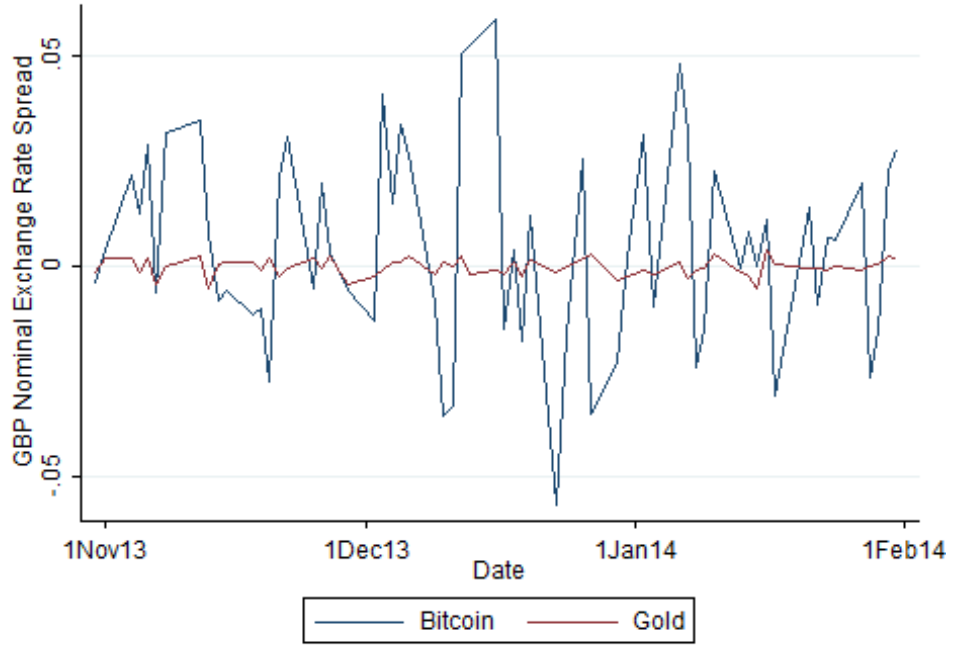


Figure 8: Implied vs. Market Rate Spread for  $\$/A\$$  Rate

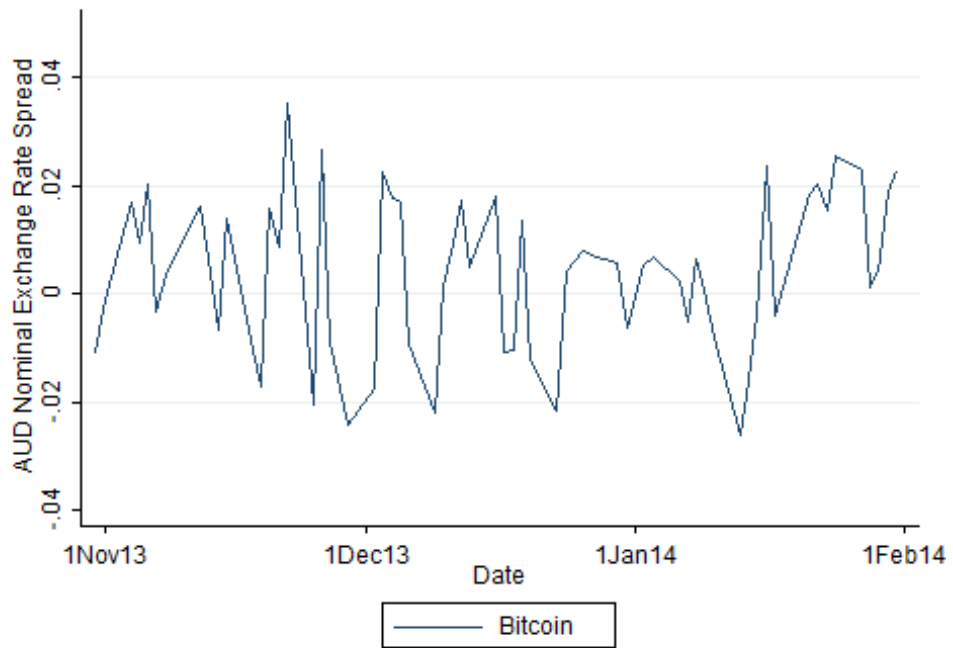




Figure 9: Bitcoin Market-to-Implied IRF for  $\$/\text{€}$  Rate

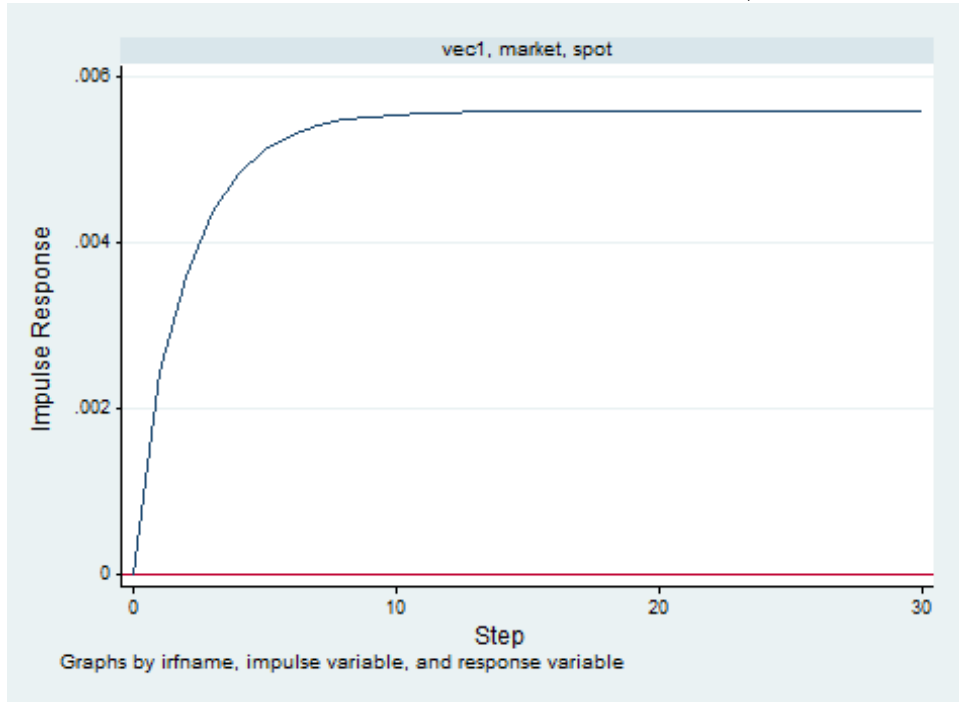


Figure 10: Bitcoin Market-to-Implied IRF for  $\$/\text{£}$  Rate

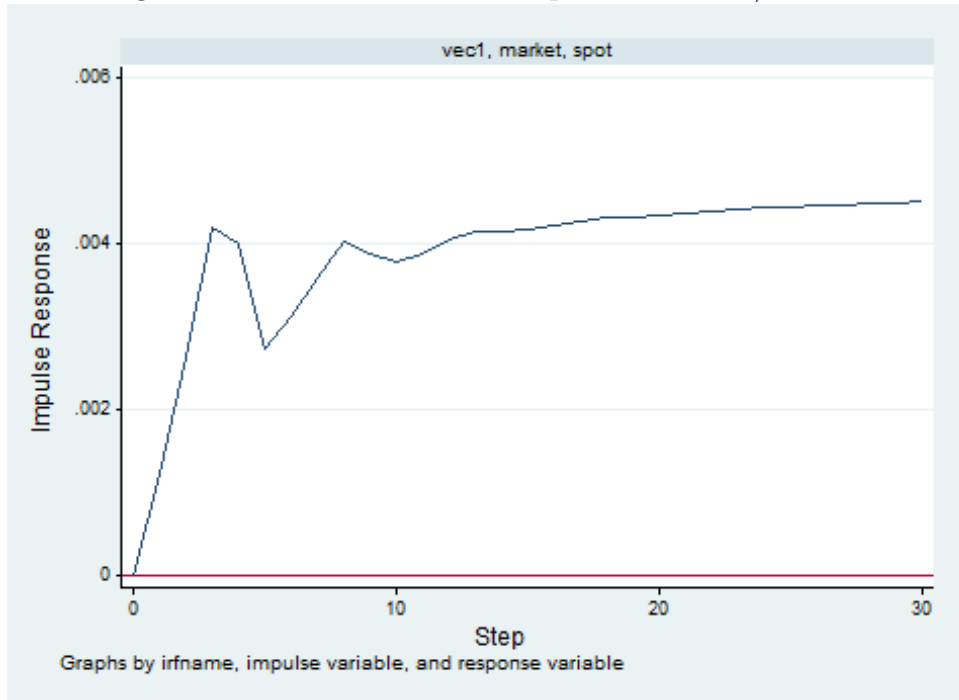


Figure 11: Bitcoin Market-to-Implied IRF for  $\$/A\$$  Rate

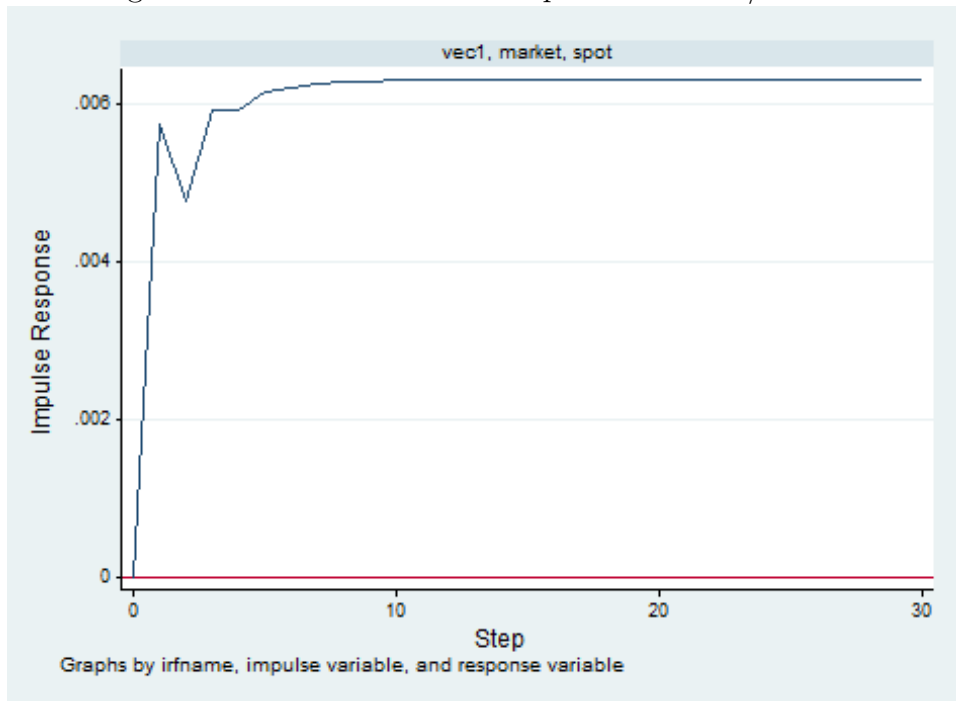


Figure 12: Bitcoin Implied-to-Market IRF for  $\$/\text{€}$  Rate

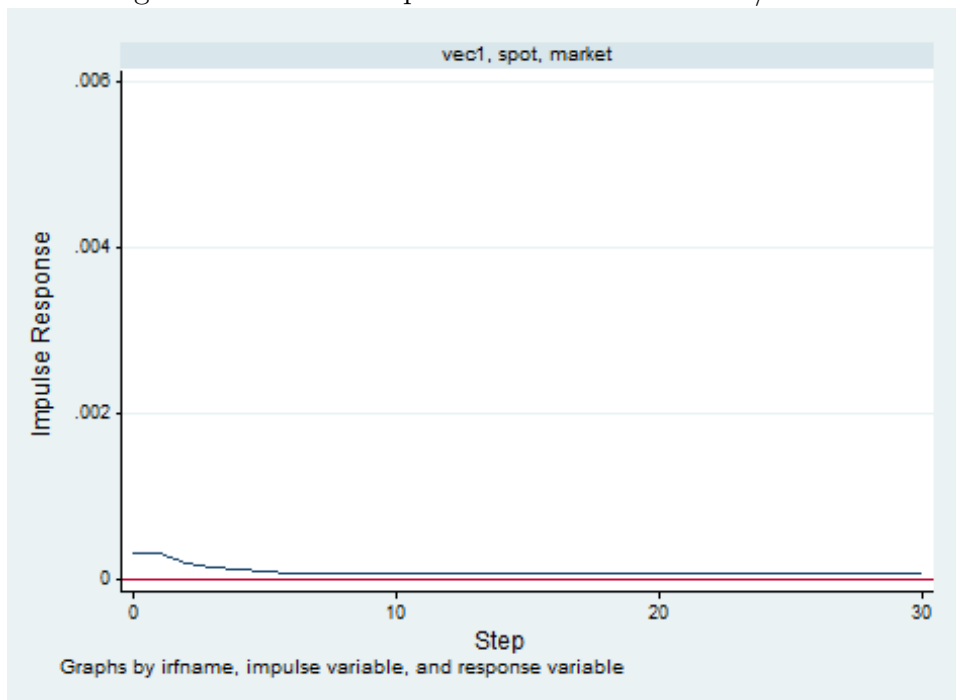


Figure 13: Bitcoin Implied-to-Market IRF for  $\$/\pounds$  Rate

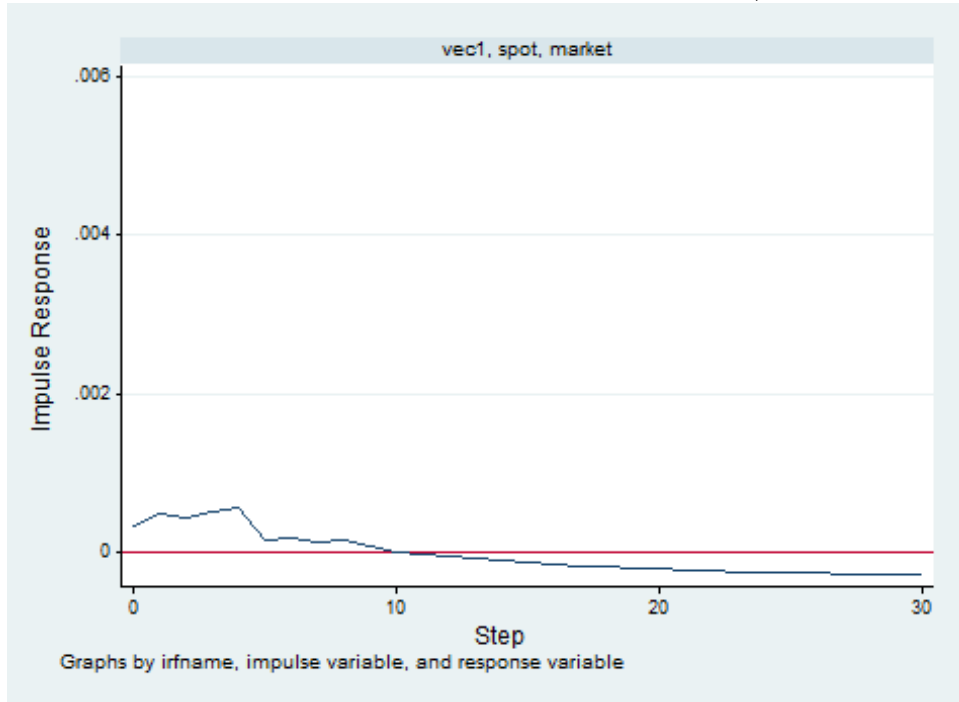


Figure 14: Bitcoin Implied-to-Market IRF for  $\$/A\$$  Rate

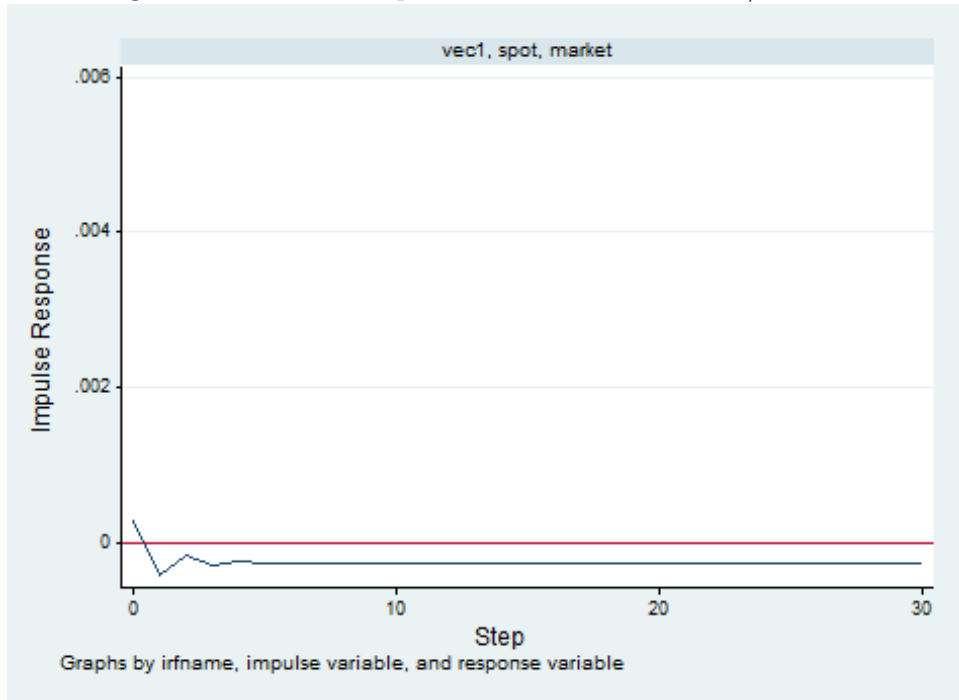


Figure 15: Gold Market-to-Implied IRF for  $\$/\text{€}$  Rate

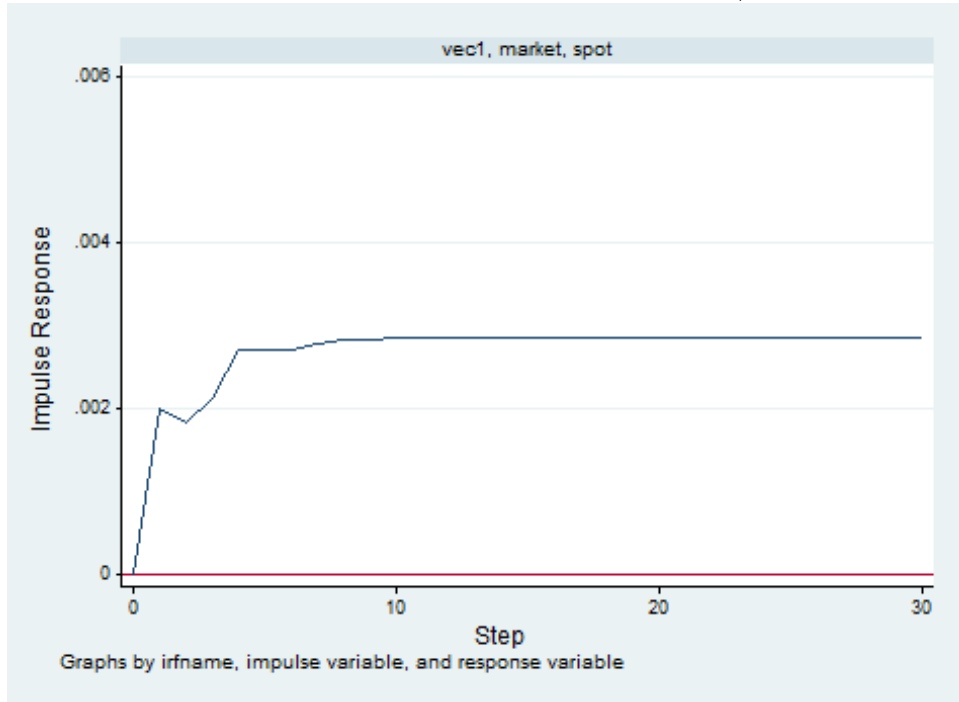


Figure 16: Gold Market-to-Implied IRF for  $\$/\text{£}$  Rate

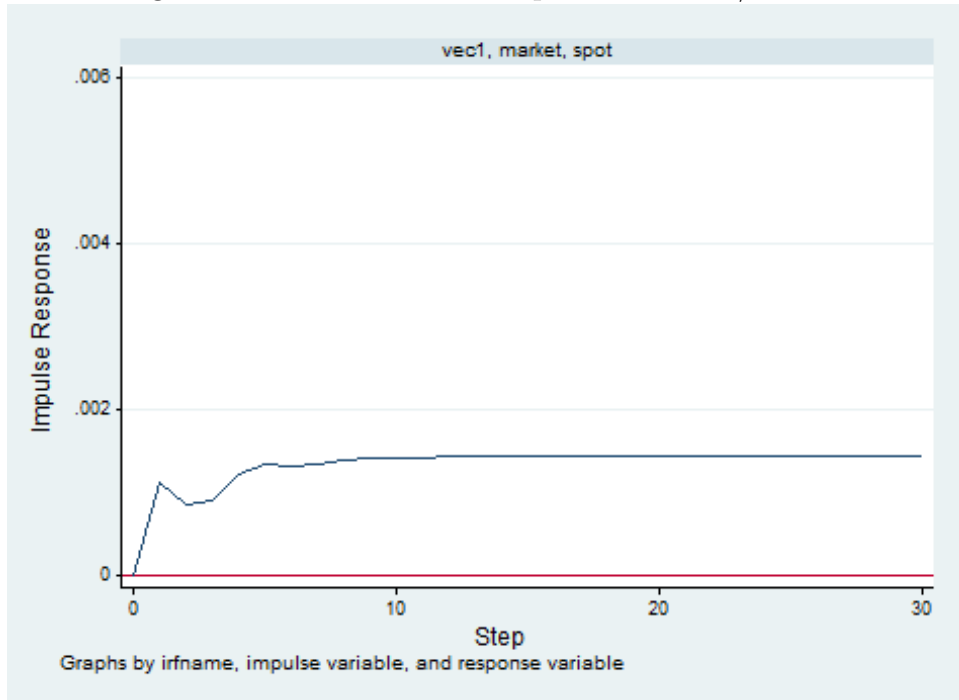


Figure 17: Gold Implied-to-Market IRF for  $\$/\text{€}$  Rate

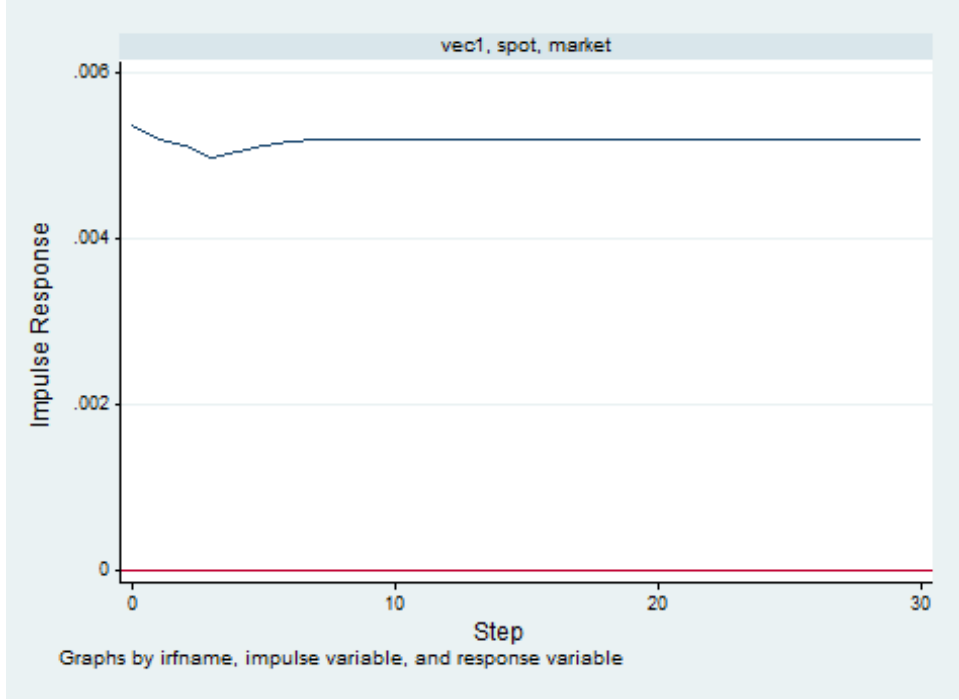


Figure 18: Gold Implied-to-Market IRF for  $\$/\text{£}$  Rate

